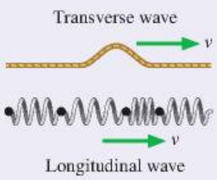
16 Traveling Waves

This surfer is "catching a wave." At the same time, he's seeing light waves and hearing sound waves.

IN THIS CHAPTER, you will learn the basic properties of traveling waves.

What is a wave?

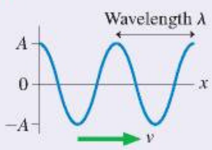
A wave is a disturbance traveling through a medium. In a transverse wave, the displacement is perpendicular to the direction of travel. In a longitudinal wave, the displacement is parallel to the direction of travel.



What are some wave properties?

A wave is characterized by:

- Wave speed: How fast it travels through the medium.
- Wavelength: The distance between two neighboring crests.
- Frequency: The number of oscillations per second.
- Amplitude: The maximum displacement.

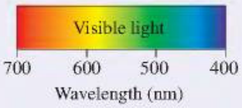


« LOOKING BACK Sections 15.1–15.2 Properties of simple harmonic motion

Are sound and light waves?

Yes! Very important waves.

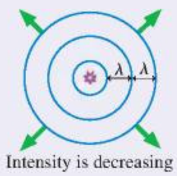
- Sound waves are longitudinal waves.
- Light waves are transverse waves.



The colors of visible light correspond to different wavelengths.

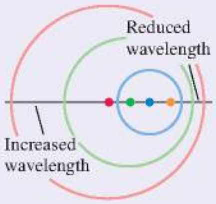
Do waves carry energy?

They do. The rate at which a wave delivers energy to a surface is the intensity of the wave. For sound waves, we'll use a logarithmic decibel scale to characterize the loudness of a sound.



What is the Doppler effect?

The frequency and wavelength of a wave are shifted if there is relative motion between the source and the observer of the waves. This is called the Doppler effect. It explains why the pitch of an ambulance siren drops as it races past you.



How will I use waves?

Waves are literally everywhere. Communications systems from radios to cell phones to fiber optics use waves. Sonar and radar and medical ultrasound use waves. Music and musical instruments are all about waves. Waves are present in the oceans, the atmosphere, and the earth. This chapter and the next will allow you to understand and work with a wide variety of waves that you may meet in your career.

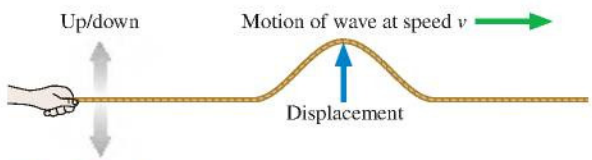
16.1 An Introduction to Waves

From sound and light to ocean waves and seismic waves, we're surrounded by waves. Understanding musical instruments, cell phones, or lasers requires a knowledge of waves. With this chapter we shift our focus from the particle model to a new **wave model** that emphasizes those aspects of wave behavior common to all waves.

The wave model is built around the idea of a **traveling wave,** which is an organized disturbance traveling with a well-defined wave speed. We'll begin our study of traveling waves by looking at two distinct wave motions.

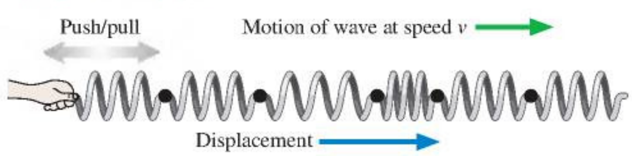
Two types of traveling waves

A transverse wave



A **transverse wave** is a wave in which the displacement is *perpendicular* to the direction in which the wave travels. For example, a wave travels along a string in a horizontal direction while the particles that make up the string oscillate vertically. Electromagnetic waves are also transverse waves because the electromagnetic fields oscillate perpendicular to the direction in which the wave travels.

A longitudinal wave



In a **longitudinal wave,** the particles in the medium are displaced *parallel* to the direction in which the wave travels. Here we see a chain of masses connected by springs. If you give the first mass in the chain a sharp push, a disturbance travels down the chain by compressing and expanding the springs. Sound waves in gases and liquids are the most well known examples of longitudinal waves.

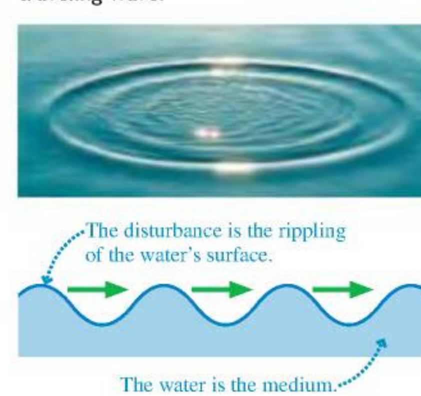
We can also classify waves on the basis of what is "waving":

1. **Mechanical waves** travel only within a material *medium,* such as air or water. Two familiar mechanical waves are sound waves and water waves.
2. **Electromagnetic waves,** from radio waves to visible light to x rays, are a self-sustaining oscillation of the *electromagnetic field.* Electromagnetic waves require no material medium and can travel through a vacuum.

The **medium** of a mechanical wave is the substance through or along which the wave moves. For example, the medium of a water wave is the water, the medium of a sound wave is the air, and the medium of a wave on a stretched string is the string. A medium must be *elastic.* That is, a restoring force of some sort brings the medium back to equilibrium after it has been displaced or disturbed. The tension in a stretched string pulls the string back straight after you pluck it. Gravity restores the level surface of a lake after the wave generated by a boat has passed by.

As a wave passes through a medium, the atoms of the medium—we'll simply call them the particles of the medium—are displaced from equilibrium. This is a **disturbance** of the medium. The water ripples of FIGURE 16.1 are a disturbance of the water's surface. A pulse traveling down a string is a disturbance, as are the wake of a boat and the sonic boom created by a jet traveling faster than the speed of sound. **The disturbance of a wave is an *organized* motion of the particles in the medium,** in contrast to the *random* molecular motions of thermal energy.

FIGURE 16.1 Ripples on a pond are a traveling wave.



Wave Speed

A wave disturbance is created by a *source.* The source of a wave might be a rock thrown into water, your hand plucking a stretched string, or an oscillating loudspeaker cone pushing on the air. Once created, the disturbance travels outward through the medium at the **wave speed** v. This is the speed with which a ripple moves across the water or a pulse travels down a string.

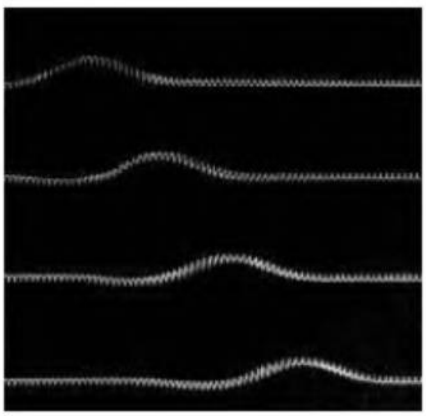
NOTE The disturbance propagates through the medium, but the medium as a whole does not move! The ripples on the pond (the disturbance) move outward from the splash of the rock, but there is no outward flow of water from the splash. **A wave transfers energy, but it does not transfer any material or substance outward from the source.**

As an example, we'll prove in Section 16.4 that the wave speed on a string stretched with tension T_s is

$$v_{string} = \sqrt{\frac{T_s}{\mu}} \quad \text{(wave speed on a stretched string)} \tag{16.1}$$

where μ is the string's **linear density,** its mass-to-length ratio:

$$\mu = \frac{m}{L} \tag{16.2}$$



This sequence of photographs shows a wave pulse traveling along a spring.

The SI unit of linear density is kg/m. A fat string has a larger value of μ than a skinny string made of the same material. Similarly, a steel wire has a larger value of μ than a plastic string of the same diameter. We'll assume that strings are *uniform,* meaning the linear density is the same everywhere along the length of the string.

NOTE The subscript s on the symbol T_s for the string's tension distinguishes it from the symbol T for the *period* of oscillation.

Equation 16.1 is the wave *speed,* not the wave velocity, so v_{string} always has a positive value. Every point on a wave travels with this speed. You can increase the wave speed either by *increasing* the string's tension (make it tighter) or by *decreasing* the string's linear density (make it skinnier).

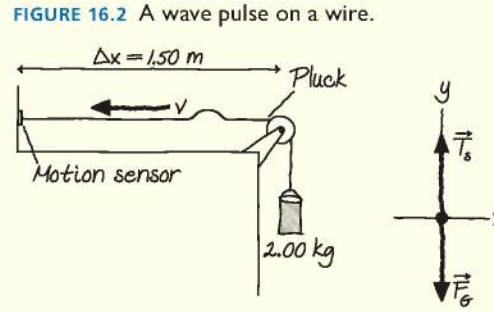
EXAMPLE 16.1 Measuring the linear density

A 2.00-m-long metal wire is attached to a motion sensor, stretched horizontally to a pulley 1.50 m away, then connected to a 2.00 kg hanging mass that provides tension. A mechanical pick plucks a horizontal segment of wire right at the pulley, creating a small wave pulse that travels along the wire. The plucking motion starts a timer that is stopped by the motion sensor when the pulse reaches the end of the wire. What is the wire's linear density if the pulse takes 18.0 ms to travel the length of the wire?

MODEL Model the pulse as a traveling wave and the pulley as frictionless.

VISUALIZE FIGURE 16.2 is a pictorial representation. The free-body diagram is for the hanging mass.

FIGURE 16.2 A wave pulse on a wire.



SOLVE The wave speed on a wire is determined by the wire's linear density μ and tension T_s. By measuring the wave speed and the tension, we can determine the linear density. The hanging mass is in equilibrium, with no net force, so we see from the free-body diagram that the tension throughout the wire (because the pulley is frictionless) is $T_s = F_G = Mg = 19.6$ N. Because the wave pulse travels 1.50 m in 18.0 ms, its speed is

$$v = \frac{1.50 \text{ m}}{0.0180 \text{ s}} = 83.3 \text{ m/s}$$

Thus, from Equation 16.1, the wire's linear density is

$$\mu = \frac{T_s}{v^2} = \frac{19.6 \text{ N}}{(83.3 \text{ m/s})^2} = 2.82 \times 10^{-3} \text{ kg/m} = 2.82 \text{ g/m}$$

Linear densities of strings are often stated in g/m, although these are not SI units. You must use kg/m in any calculations.

ASSESS A meter of thin wire is likely to have a mass of a few grams, so a linear density of a few g/m is reasonable. Note that the total length of the wire was not relevant.

The wave speed on a string is a property of the string—its tension and linear density. In general, **the wave speed is a property *of the medium.*** The wave speed depends on the restoring forces within the medium but not at all on the shape or size of the pulse, how the pulse was generated, or how far it has traveled.

STOP TO THINK 16.1 Which of the following actions would make a pulse travel faster along a stretched string? More than one answer may be correct. If so, give all that are correct.

a. Move your hand up and down more quickly as you generate the pulse.
b. Move your hand up and down a larger distance as you generate the pulse.
c. Use a heavier string of the same length, under the same tension.
d. Use a lighter string of the same length, under the same tension.
e. Stretch the string tighter to increase the tension.
f. Loosen the string to decrease the tension.
g. Put more force into the wave.

16.2 One-Dimensional Waves

To understand waves we must deal with functions of *two* variables. Until now, we have been concerned with quantities that depend only on time, such as $x(t)$ or $v(t)$. Functions of the one variable t are appropriate for a particle because a particle is only in one place at a time, but a wave is not localized. It is spread out through space at each instant of time. To describe a wave mathematically requires a function that specifies not only an instant of time (when) but also a point in space (where).

Rather than leaping into mathematics, we will start by thinking about waves graphically. Consider the wave pulse shown moving along a stretched string in **FIGURE 16.3**. (We will consider somewhat artificial triangular and square-shaped pulses in this section to make clear where the edges of the pulse are.) The graph shows the string's displacement Δy at a particular instant of time t_1 as a function of position x along the string. This is a "snapshot" of the wave, much like what you might make with a camera whose shutter is opened briefly at t_1. A graph that shows the wave's displacement as a function of position at a single instant of time is called a **snapshot graph.** For a wave on a string, a snapshot graph is literally a picture of the wave at this instant.

FIGURE 16.4 shows a sequence of snapshot graphs as the wave of Figure 16.3 continues to move. These are like successive frames from a video. Notice that the wave pulse moves forward distance $\Delta x = v\,\Delta t$ during the time interval Δt. That is, the wave moves with constant speed.

A snapshot graph tells only half the story. It tells us *where* the wave is and how it varies with position, but only at one instant of time. It gives us no information about how the wave *changes* with time. As a different way of portraying the wave, suppose we follow the dot marked on the string in Figure 16.4 and produce a graph showing how the displacement of this dot changes with time. The result, shown in **FIGURE 16.5**, is a displacement-versus-time graph at a single position in space. A graph that shows the wave's displacement as a function of time at a single position in space is called a **history graph.** It tells the history of that particular point in the medium.

FIGURE 16.3 A snapshot graph of a wave pulse on a string.

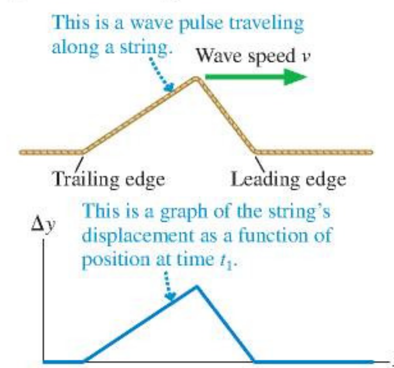


FIGURE 16.4 A sequence of snapshot graphs shows the wave in motion.

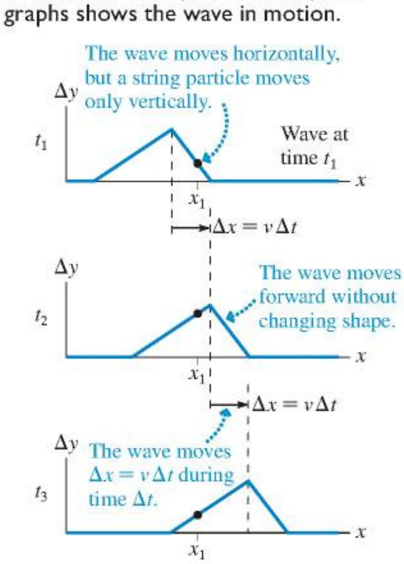


FIGURE 16.5 A history graph for the dot on the string in Figure 16.4.

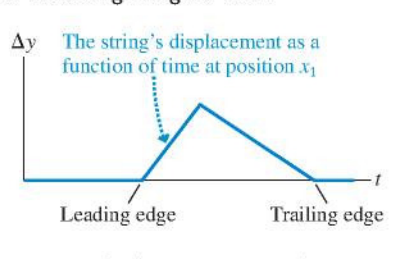
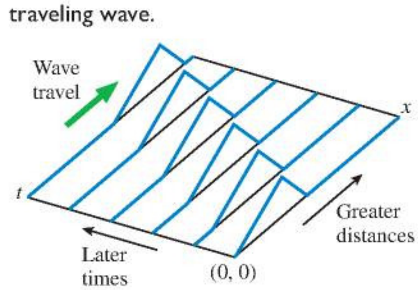


FIGURE 16.6 An alternative look at a traveling wave.



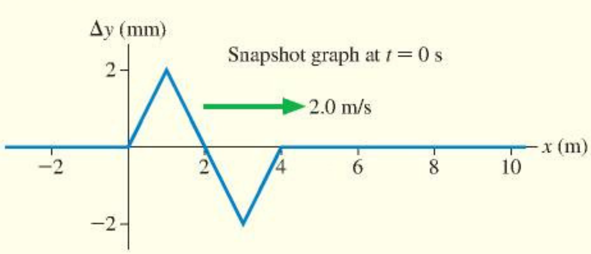
You might think we have made a mistake; the graph of Figure 16.5 is reversed compared to Figure 16.4. It is not a mistake, but it requires careful thought to see why. As the wave moves toward the dot, the steep **leading edge** causes the dot to rise quickly. On the displacement-versus-time graph, *earlier* times (smaller values of t) are to the *left* and later times (larger t) to the right. Thus the leading edge of the wave is on the *left* side of the Figure 16.5 history graph. As you move to the right on Figure 16.5 you see the slowly falling **trailing edge** of the wave as it moves past the dot at later times.

The snapshot graph of Figure 16.3 and the history graph of Figure 16.5 portray complementary information. The snapshot graph tells us how things look throughout all of space, but at only one instant of time. The history graph tells us how things look at all times, but at only one position in space. We need them both to have the full story of the wave. An alternative representation of the wave is the series of graphs in FIGURE 16.6, where we can get a clearer sense of the wave moving forward. But graphs like these are essentially impossible to draw by hand, so it is necessary to move back and forth between snapshot graphs and history graphs.

EXAMPLE 16.2 Finding a history graph from a snapshot graph

FIGURE 16.7 is a snapshot graph at $t = 0$ s of a wave moving to the right at a speed of 2.0 m/s. Draw a history graph for the position $x = 8.0$ m.

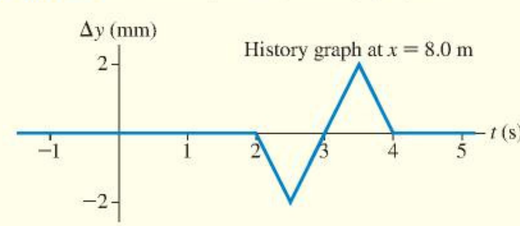
FIGURE 16.7 A snapshot graph at $t = 0$ s.



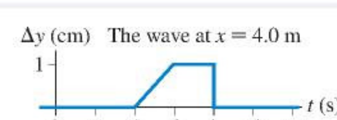
MODEL This is a wave traveling at constant speed. The pulse moves 2.0 m to the right every second.

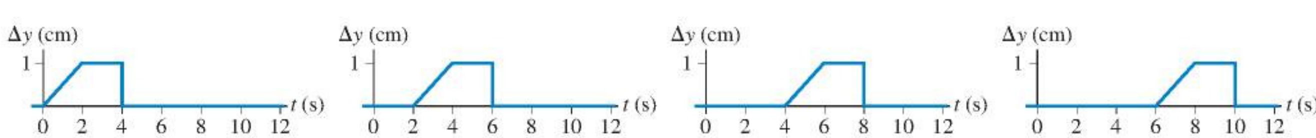
VISUALIZE The snapshot graph of Figure 16.7 shows the wave at all points on the x-axis at $t = 0$ s. You can see that nothing is happening at $x = 8.0$ m at this instant of time because the wave has not yet reached $x = 8.0$ m. In fact, at $t = 0$ s the leading edge of the wave is still 4.0 m away from $x = 8.0$ m. Because the wave is traveling at 2.0 m/s, it will take 2.0 s for the leading edge to reach $x = 8.0$ m. Thus the history graph for $x = 8.0$ m will be zero until $t = 2.0$ s. The first part of the wave causes a *downward* displacement of the medium, so immediately after $t = 2.0$ s the displacement at $x = 8.0$ m will be negative. The negative portion of the wave pulse is 2.0 m wide and takes 1.0 s to pass $x = 8.0$ m, so the midpoint of the pulse reaches $x = 8.0$ m at $t = 3.0$ s. The positive portion takes another 1.0 s to go past, so the trailing edge of the pulse arrives at $t = 4.0$ s. You could also note that the trailing edge was initially 8.0 m away from $x = 8.0$ m and needed 4.0 s to travel that distance at 2.0 m/s. The displacement at $x = 8.0$ m returns to zero at $t = 4.0$ s and remains zero for all later times. This information is all portrayed on the history graph of FIGURE 16.8.

FIGURE 16.8 The corresponding history graph at $x = 8.0$ m.



STOP TO THINK 16.2 The graph at the right is the history graph at $x = 4.0$ m of a wave traveling to the right at a speed of 2.0 m/s. Which is the history graph of this wave at $x = 0$ m?





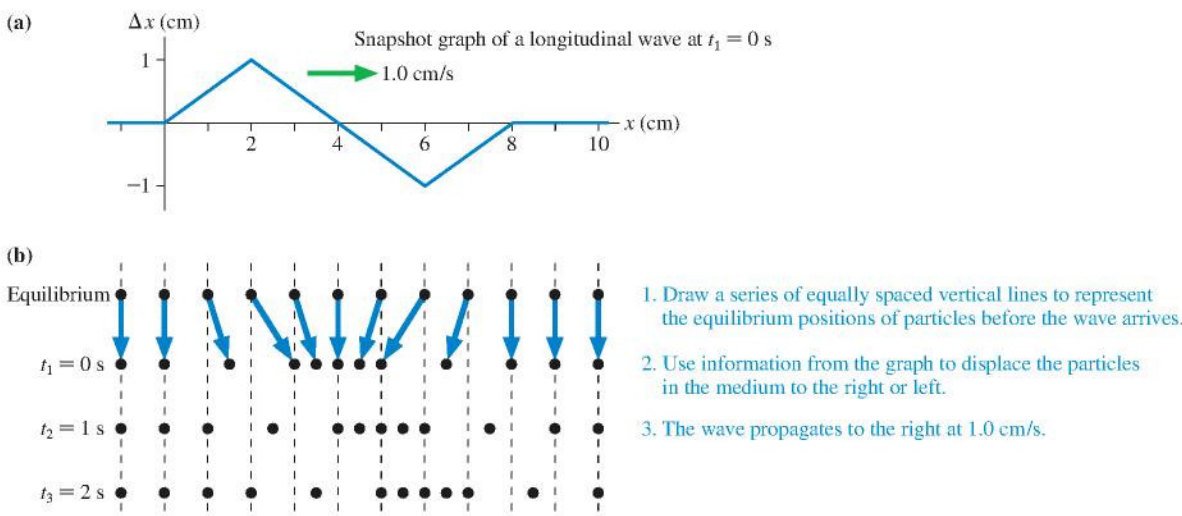
(a) (b) (c) (d)

Longitudinal Waves

For a wave on a string, a transverse wave, the snapshot graph is literally a picture of the wave. Not so for a longitudinal wave, where the particles in the medium are displaced parallel to the direction in which the wave is traveling. Thus the displacement is Δx rather than Δy, and a snapshot graph is a graph of Δx versus x.

FIGURE 16.9a is a snapshot graph of a longitudinal wave, such as a sound wave. It's purposefully drawn to have the same shape as the string wave in Example 16.2. Without practice, it's not clear what this graph tells us about the particles in the medium.

FIGURE 16.9 Visualizing a longitudinal wave.



To help you find out, **FIGURE 16.9b** provides a tool for visualizing longitudinal waves. In the second row, we've used information from the graph to displace the particles in the medium to the right or to the left of their equilibrium positions. For example, the particle at $x = 1.0$ cm has been displaced 0.5 cm to the right because the snapshot graph shows $\Delta x = 0.5$ cm at $x = 1.0$ cm. We now have a picture of the longitudinal wave pulse at $t_1 = 0$ s. You can see that the medium is compressed to higher density at the center of the pulse and, to compensate, expanded to lower density at the leading and trailing edges. Two more lines show the medium at $t_2 = 1$ s and $t_3 = 2$ s so that you can see the wave propagating through the medium at 1.0 cm/s.

The Displacement

A traveling wave causes the particles of the medium to be displaced from their equilibrium positions. Because one of our goals is to develop a mathematical representation to describe all types of waves, we'll use the generic symbol D to stand for the *displacement* of a wave of any type. But what do we mean by a "particle" in the medium? And what about electromagnetic waves, for which there is no medium?

For a string, where the atoms stay fixed relative to each other, you can think of either the atoms themselves or very small segments of the string as being the particles of the medium. D is then the perpendicular displacement Δy of a point on the string. For a sound wave, D is the longitudinal displacement Δx of a small volume of fluid. For any other mechanical wave, D is the appropriate displacement. Even electromagnetic waves can be described within the same mathematical representation if D is interpreted as a yet-undefined *electromagnetic field strength,* a "displacement" in a more abstract sense as an electromagnetic wave passes through a region of space.

You've probably seen or participated in "the wave" at a sporting event. The wave moves around the stadium, but the people (the medium) simply undergo small displacements from their equilibrium positions.

Because the displacement of a particle in the medium depends both on *where* the particle is (position x) and on *when* you observe it (time t), D must be a function of the two variables x and t. That is,

$$D(x, t) = \text{the displacement at time } t \text{ of a particle at position } x$$

The values of *both* variables—where and when—must be specified before you can evaluate the displacement D.

16.3 Sinusoidal Waves

A wave source that oscillates with simple harmonic motion (SHM) generates a **sinusoidal wave.** For example, a loudspeaker cone that oscillates in SHM radiates a sinusoidal sound wave. The sinusoidal electromagnetic waves broadcast by television and FM radio stations are generated by electrons oscillating back and forth in the antenna wire with SHM. **The frequency f of the wave is the frequency of the oscillating source.**

FIGURE 16.10 A sinusoidal wave moving along the x-axis.

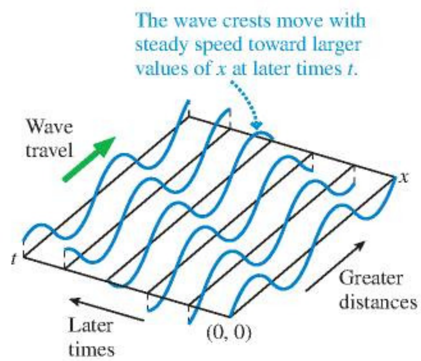
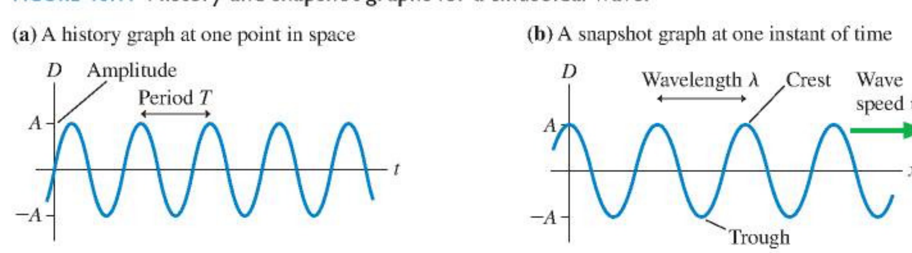


FIGURE 16.10 shows a sinusoidal wave moving through a medium. To understand how this wave propagates, let's look at history and snapshot graphs. FIGURE 16.11a is a history graph, showing the displacement of the medium at one point in space. Each particle in the medium undergoes simple harmonic motion with frequency f, so this graph of SHM is identical to the graphs you learned to work with in Chapter 15. The *period* of the wave, shown on the graph, is the time interval for one cycle of the motion. The period is related to the wave frequency f by

$$T = \frac{1}{f} \tag{16.3}$$

exactly as in simple harmonic motion. The **amplitude** A of the wave is the maximum value of the displacement. The crests of the wave have displacement $D_{crest} = A$ and the troughs have displacement $D_{trough} = -A$.

FIGURE 16.11 History and snapshot graphs for a sinusoidal wave.



Displacement versus time is only half the story. FIGURE 16.11b shows a snapshot graph for the same wave at one instant in time. Here we see the wave stretched out in space, moving to the right with speed v. An important characteristic of a sinusoidal wave is that it is *periodic in space* as well as in time. As you move from left to right along the "frozen" wave in the snapshot graph, the disturbance repeats itself over and over. The distance spanned by one cycle of the motion is called the **wavelength** of the wave. Wavelength is symbolized by λ (lowercase Greek lambda) and, because it is a length, it is measured in units of meters. The wavelength is shown in Figure 16.11b as the distance between two crests, but it could equally well be the distance between two troughs.

NOTE Wavelength is the spatial analog of period. The period T is the *time* in which the disturbance at a single point in space repeats itself. The wavelength λ is the *distance* in which the disturbance at one instant of time repeats itself.

The Fundamental Relationship for Sinusoidal Waves

There is an important relationship between the wavelength and the period of a wave. FIGURE 16.12 shows this relationship through five snapshot graphs of a sinusoidal wave at time increments of one-quarter of the period T. One full period has elapsed between the first graph and the last, which you can see by observing the motion at a fixed point on the x-axis. Each point in the medium has undergone exactly one complete oscillation.

The critical observation is that the wave crest marked by an arrow has moved one full wavelength between the first graph and the last. That is, **during a time interval of exactly one period T, each crest of a sinusoidal wave travels forward a distance of exactly one wavelength λ**. Because speed is distance divided by time, the wave speed must be

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} \qquad (16.4)$$

Because $f = 1/T$, it is customary to write Equation 16.4 in the form

$$v = \lambda f \qquad (16.5)$$

Although Equation 16.5 has no special name, it is *the* fundamental relationship for periodic waves. When using it, keep in mind the *physical* meaning that a wave moves forward a distance of one wavelength during a time interval of one period.

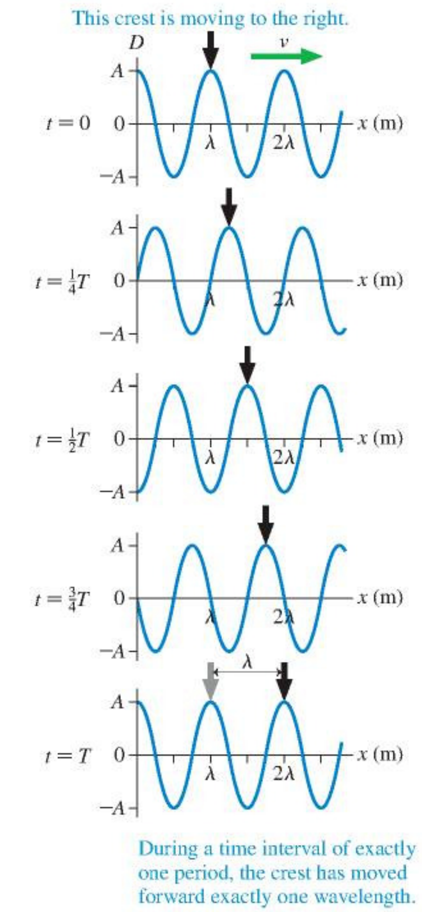
NOTE Wavelength and period are defined only for *periodic* waves, so Equations 16.4 and 16.5 apply only to periodic waves. A wave pulse has a wave speed, but it doesn't have a wavelength or a period. Hence Equations 16.4 and 16.5 cannot be applied to wave pulses.

Because the wave speed is a property of the medium while the wave frequency is a property of the oscillating source, it is often useful to write Equation 16.5 as

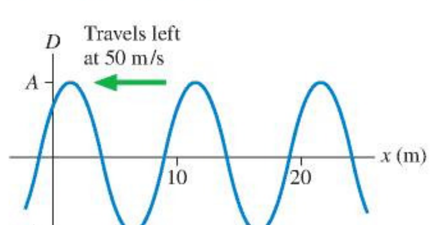
$$\lambda = \frac{v}{f} = \frac{\text{property of the medium}}{\text{property of the source}} \qquad (16.6)$$

The wavelength is a *consequence* of a wave of frequency f traveling through a medium in which the wave speed is v.

FIGURE 16.12 A series of snapshot graphs at time increments of one-quarter of the period T.



STOP TO THINK 16.3 What is the frequency of this traveling wave?



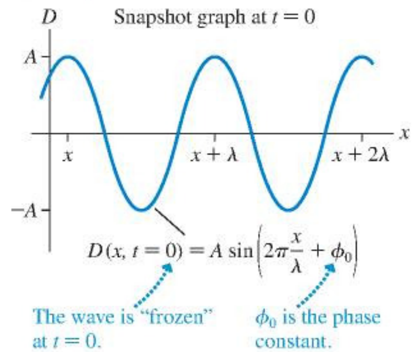
a. 0.1 Hz
b. 0.2 Hz
c. 2 Hz
d. 5 Hz
e. 10 Hz
f. 500 Hz

The Mathematics of Sinusoidal Waves

FIGURE 16.13 on the next page shows a snapshot graph at $t = 0$ of a sinusoidal wave. The sinusoidal function that describes the displacement of this wave is

$$D(x, t = 0) = A \sin\left(2\pi \frac{x}{\lambda} + \phi_0\right) \qquad (16.7)$$

FIGURE 16.13 A sinusoidal wave.



where the notation $D(x, t = 0)$ means that we've frozen the time at $t = 0$ to make the displacement a function of only x. The term ϕ_0 is a *phase constant* that characterizes the initial conditions. (We'll return to the phase constant momentarily.)

The function of Equation 16.7 is periodic with period λ. We can see this by writing

$$D(x + \lambda) = A \sin\left(2\pi \frac{(x + \lambda)}{\lambda} + \phi_0\right) = A \sin\left(2\pi \frac{x}{\lambda} + \phi_0 + 2\pi \text{ rad}\right)$$

$$= A \sin\left(2\pi \frac{x}{\lambda} + \phi_0\right) = D(x)$$

where we used the fact that $\sin(a + 2\pi \text{ rad}) = \sin a$. In other words, the disturbance created by the wave at $x + \lambda$ is exactly the same as the disturbance at x.

The next step is to set the wave in motion. We can do this by replacing x in Equation 16.7 with $x - vt$. To see why this works, recall that the wave moves distance vt during time t. In other words, whatever displacement the wave has at position x at time t, the wave must have had that same displacement at position $x - vt$ at the earlier time $t = 0$. Mathematically, this idea can be captured by writing

$$D(x, t) = D(x - vt, t = 0) \tag{16.8}$$

Make sure you understand how this statement describes a wave moving in the positive x-direction at speed v.

This is what we were looking for. $D(x, t)$ is the general function describing the traveling wave. It's found by taking the function that describes the wave at $t = 0$—the function of Equation 16.7—and replacing x with $x - vt$. Thus the displacement equation of a sinusoidal wave traveling in the positive x-direction at speed v is

$$D(x, t) = A \sin\left(2\pi \frac{x - vt}{\lambda} + \phi_0\right) = A \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi_0\right) \tag{16.9}$$

In the last step we used $v = \lambda f = \lambda/T$ to write $v/\lambda = 1/T$. The function of Equation 16.9 is not only periodic in space with period λ, it is also periodic in time with period T. That is, $D(x, t + T) = D(x, t)$.

It will be useful to introduce two new quantities. First, recall from simple harmonic motion the *angular frequency*

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{16.10}$$

The units of ω are rad/s, although many textbooks use simply s^{-1}.

You can see that ω is 2π times the reciprocal of the period in time. This suggests that we define an analogous quantity, called the **wave number** k, that is 2π times the reciprocal of the period in space:

$$k = \frac{2\pi}{\lambda} \tag{16.11}$$

The units of k are rad/m, although many textbooks use simply m^{-1}.

NOTE The wave number k is *not* a spring constant, even though it uses the same symbol. This is a most unfortunate use of symbols, but every major textbook and professional tradition uses the same symbol k for these two very different meanings, so we have little choice but to follow along.

We can use the fundamental relationship $v = \lambda f$ to find an analogous relationship between ω and k:

$$v = \lambda f = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k} \tag{16.12}$$

which is usually written

$$\omega = vk \tag{16.13}$$

Equation 16.13 contains no new information. It is a variation of Equation 16.5, but one that is convenient when working with k and ω.

If we use the definitions of Equations 16.10 and 16.11, Equation 16.9 for the displacement can be written

$$D(x, t) = A\sin(kx - \omega t + \phi_0) \tag{16.14}$$
(sinusoidal wave traveling in the positive x-direction)

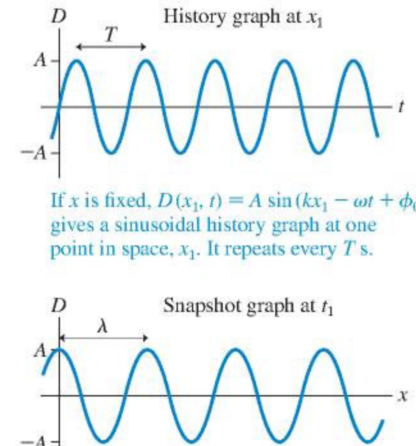
A sinusoidal wave traveling in the negative x-direction is $A\sin(kx + \omega t + \phi_0)$. Equation 16.14 is graphed versus x and t in **FIGURE 16.14**.

You learned in « Section 15.2 that the initial conditions of an oscillator can be characterized by a phase constant. The same is true for a sinusoidal wave. At $(x, t) = (0\text{ m}, 0\text{ s})$ Equation 16.14 becomes

$$D(0\text{ m}, 0\text{ s}) = A\sin\phi_0 \tag{16.15}$$

Different values of ϕ_0 describe different initial conditions for the wave.

FIGURE 16.14 Interpreting the equation of a sinusoidal traveling wave.



If x is fixed, $D(x_1, t) = A\sin(kx_1 - \omega t + \phi_0)$ gives a sinusoidal history graph at one point in space, x_1. It repeats every T s.

If t is fixed, $D(x, t_1) = A\sin(kx - \omega t_1 + \phi_0)$ gives a sinusoidal snapshot graph at one instant of time, t_1. It repeats every λ m.

EXAMPLE 16.3 Analyzing a sinusoidal wave

A sinusoidal wave with an amplitude of 1.00 cm and a frequency of 100 Hz travels at 200 m/s in the positive x-direction. At $t = 0$ s, the point $x = 1.00$ m is on a crest of the wave.

a. Determine the values of A, v, λ, k, f, ω, T, and ϕ_0 for this wave.
b. Write the equation for the wave's displacement as it travels.
c. Draw a snapshot graph of the wave at $t = 0$ s.

VISUALIZE The snapshot graph will be sinusoidal, but we must do some numerical analysis before we know how to draw it.

SOLVE a. There are several numerical values associated with a sinusoidal traveling wave, but they are not all independent. From the problem statement itself we learn that

$$A = 1.00\text{ cm} \qquad v = 200\text{ m/s} \qquad f = 100\text{ Hz}$$

We can then find:

$$\lambda = v/f = 2.00\text{ m}$$
$$k = 2\pi/\lambda = \pi\text{ rad/m or } 3.14\text{ rad/m}$$
$$\omega = 2\pi f = 628\text{ rad/s}$$
$$T = 1/f = 0.0100\text{ s} = 10.0\text{ ms}$$

The phase constant ϕ_0 is determined by the initial conditions. We know that a wave crest, with displacement $D = A$, is passing $x_0 = 1.00$ m at $t_0 = 0$ s. Equation 16.14 at x_0 and t_0 is

$$D(x_0, t_0) = A = A\sin[k(1.00\text{ m}) + \phi_0]$$

This equation is true only if $\sin[k(1.00\text{ m}) + \phi_0] = 1$, which requires

$$k(1.00\text{ m}) + \phi_0 = \frac{\pi}{2}\text{ rad}$$

Solving for the phase constant gives

$$\phi_0 = \frac{\pi}{2}\text{ rad} - (\pi\text{ rad/m})(1.00\text{ m}) = -\frac{\pi}{2}\text{ rad}$$

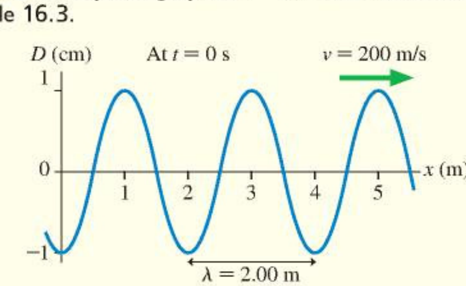
b. With the information gleaned from part a, the wave's displacement is

$$D(x, t) = 1.00\text{ cm} \times \sin[(3.14\text{ rad/m})x - (628\text{ rad/s})t - \pi/2\text{ rad}]$$

Notice that we included units with A, k, ω, and ϕ_0.

c. We know that $x = 1.00$ m is a wave crest at $t = 0$ s and that the wavelength is $\lambda = 2.00$ m. Because the origin is $\lambda/2$ away from the crest at $x = 1.00$ m, we expect to find a wave trough at $x = 0$. This is confirmed by calculating $D(0\text{ m}, 0\text{ s}) = (1.00\text{ cm})\sin(-\pi/2\text{ rad}) = -1.00$ cm. **FIGURE 16.15** is a snapshot graph that portrays this information.

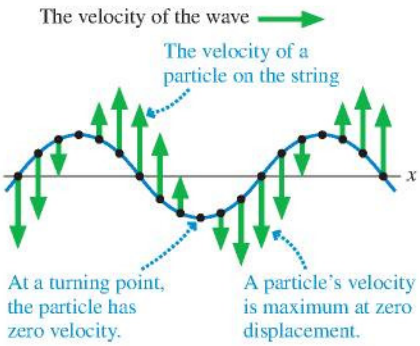
FIGURE 16.15 A snapshot graph at $t = 0$ s of the sinusoidal wave of Example 16.3.



The Velocity of a Particle in the Medium

As a sinusoidal wave travels along the x-axis with speed v, the particles of the medium oscillate back and forth in SHM. For a transverse wave, such as a wave on a string, the oscillation is in the y-direction. For a longitudinal sound wave, the particles oscillate

FIGURE 16.16 A snapshot graph of a wave on a string with vectors showing the velocity *of the string* at various points.



in the x-direction, parallel to the propagation. We can use the displacement equation, Equation 16.14, to find the velocity of a particle in the medium.

At time t, the displacement of the medium at position x is

$$D(x, t) = A\sin(kx - \omega t + \phi_0) \tag{16.16}$$

The velocity of the medium—**which is not the same as the velocity of the wave along the string**—is the time derivative of Equation 16.16:

$$v = \frac{dD}{dt} = -\omega A\cos(kx - \omega t + \phi_0) \tag{16.17}$$

Thus the maximum speed of particles in the medium is $v_{max} = \omega A$. This is the same result we found for simple harmonic motion because the motion of the medium *is* simple harmonic motion. **FIGURE 16.16** shows velocity vectors *of the particles* at different points along a string as a sinusoidal wave moves from left to right.

NOTE Creating a wave of larger amplitude increases the speed of particles in the medium, but it does *not* change the speed of the wave *through* the medium.

16.4 ADVANCED TOPIC The Wave Equation on a String

Why do waves propagate along a string? We've described wave motion—the kinematics of waves—but not explained why it occurs. The motion of a string, like that of a baseball, is governed by Newton's second law. But a baseball can be modeled as a moving particle. To explain waves, we need to see how Newton's laws apply to a *continuous* object that is spread out in space.

This section will be significantly more mathematical than any analysis that we've done so far, so it will be good to get an overview of where we're going. We have two primary goals:

- To use Newton's second law to find an *equation of motion* for displacements on a string. This is called the *wave equation*. We'll see that Equation 16.14, the displacement of a sinusoidal wave, is a solution to the wave equation.
- To predict the wave speed on a string.

Although we'll derive the wave equation for a string, the equation itself occurs in many other contexts in science and engineering. Wherever this equation arises, the solutions are traveling waves.

FIGURE 16.17 Apply Newton's second law to this small piece of string.

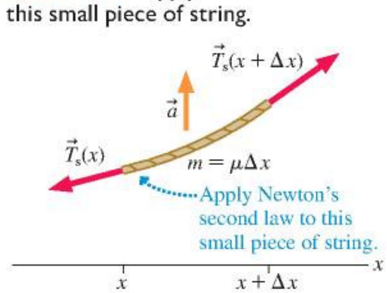


FIGURE 16.17 shows a small piece of string that is displaced from its equilibrium position. This piece is at position x and has a small horizontal width Δx. We're going to apply Newton's second law, the familiar $F_{net} = ma$, to this little piece of string. Notice that it's curved, so the tension forces at the ends are *not* opposite each other. This is essential in order for there to be a net force.

We'll begin by making the realistic assumption that the wave amplitude A is much smaller than the wavelength λ. On average, the string "rises" distance A over a "run" of $\lambda/4$. If $A \ll \lambda$, then the slope of the string is always very small. That is, the string itself and the tension vectors are always very close to being horizontal. (Our drawings greatly exaggerate the amplitude for clarity.)

This assumption has two immediate implications. First, a small-amplitude wave doesn't noticeably increase the length of the string. With no additional stretching, the string tension T_s is not altered by the wave. Second, because the string is always very close to being horizontal, there's virtually no difference between the actual length of the piece of string in Figure 16.17 and its horizontal width Δx. Thus the mass of this little piece of string is $m = \mu\,\Delta x$, where, you'll recall, μ is the string's linear density (mass per unit length).

Because we know the mass, let's start with the ma side of Newton's second law. For a transverse wave, this little piece of string oscillates perpendicular to the direction of wave propagation. For wave motion along the x-axis, the string accelerates in the y-direction. If we had *only* this little piece of string, not an entire string, we would model it as a particle and write its acceleration as $a_y = dv_y/dt = d^2y/dt^2$. The acceleration of a particle, as you've learned, is the second derivative of its position with respect to time.

But the string isn't a particle. At any instant of time, different pieces of the string have different accelerations. To find the acceleration at a specific point on the string, we want to know how the displacement varies with t at that specific value of x. Or, because the displacement $D(x, t)$ is a function of two variables, we want to know the rate at which $D(x, t)$ changes with respect to t at a specific value of x. In multivariable calculus, the rate of change of a function with respect to one variable while all other variables are held fixed is called a **partial derivative.** Partial, because by holding other variables constant we're only partially examining the many ways in which the function could change.

Partial derivatives have a special notation, using a "curly d." The velocity of our little piece of string is written

$$v_y = \frac{\partial D}{\partial t} \tag{16.18}$$

and its acceleration is

$$a_y = \frac{\partial^2 D}{\partial t^2} \tag{16.19}$$

Don't panic if you've not reached partial derivatives in calculus. They are evaluated exactly like regular derivatives, but the partial-derivative notation means "treat all the other variables as if they were constants." Using the partial derivative, we find the first half of Newton's second law for our little piece of string is

$$ma_y = \mu\,\Delta x \frac{\partial^2 D}{\partial t^2} \tag{16.20}$$

Now we can turn our attention to finding the net force on this little piece of string. The type of analysis we're going to do may be new to you, but it is widely used in more advanced science and engineering courses. FIGURE 16.18 shows our little piece of the string, this time with the tension forces—one at each end of the string—resolved into x- and y-components.

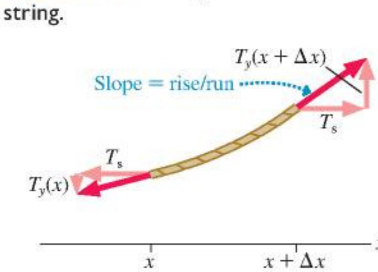


FIGURE 16.18 Finding the net force on the string.

Strictly speaking, the tension force T_s is tangent to the string. However, our small-amplitude assumption, requiring this piece of string to be almost horizontal, means there is virtually no distinction between T_s and its horizontal component. (This is the small-angle approximation $\cos\theta \approx 1$ if $\theta \ll 1$ rad.) Thus we've identified the two horizontal components as T_s. Because they are equal but opposite, the net horizontal force is zero. This has to be true because each piece of this transverse wave accelerates only in the y-direction.

The net force on this little piece of string in the transverse direction (the y-direction) is

$$F_{\text{net } y} = T_y(x + \Delta x) + T_y(x) \tag{16.21}$$

The notation $T_y(x)$ means "the y-component of the string tension at position x on the string." And we're adding, not subtracting, because this is a formal statement that the net force is the sum of all forces.

You can see, from the force triangle in Figure 16.18, that the ratio $T_y(x + \Delta x)/T_s$—rise over run—is the *slope of the string* at position $x + \Delta x$. This is a key part of the analysis, so make sure you understand it. The slope is the derivative of the string's displacement with respect to x *at this specific instant of time*. We're holding t constant while looking at the spatial variation of the string, so this is another partial derivative:

$$\text{string slope} = \frac{\partial D}{\partial x} \tag{16.22}$$

At the right end of this little piece of string, at position $x + \Delta x$, the y-component of the tension is

$$T_y(x + \Delta x) = (\text{slope at } x + \Delta x) \times T_s = T_s \left.\frac{\partial D}{\partial x}\right|_{x+\Delta x} \tag{16.23}$$

where the subscript on the partial derivative means to evaluate the slope at $x + \Delta x$. The same analysis holds at the left end, position x, with one change: Because T_s points toward the left, a "negative run," the ratio $T_y(x)/T_s$ is the *negative* of the string slope. Thus

$$T_y(x) = -(\text{slope at } x) \times T_s = -T_s \left.\frac{\partial D}{\partial x}\right|_{x} \tag{16.24}$$

Combining Equations 16.23 and 16.24, we find that the net force on this little piece of string is

$$F_{\text{net } y} = T_y(x + \Delta x) + T_y(x) = T_s \left[\left.\frac{\partial D}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial D}{\partial x}\right|_{x} \right] \tag{16.25}$$

If this little piece of string were straight, the two slopes would be the same and there would be no net force. As we noted above, the string *has* to have a curvature to have a net force.

We're almost done. We know the net force on this little piece of string (Equation 16.25) and we know its mass and acceleration (Equation 16.20). Because $F_{\text{net } y} = ma_y$, we can equate these two results:

$$T_s \left[\left.\frac{\partial D}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial D}{\partial x}\right|_{x} \right] = \mu \, \Delta x \frac{\partial^2 D}{\partial t^2} \tag{16.26}$$

Dividing by $\mu \, \Delta x$, we have

$$\frac{\partial^2 D}{\partial t^2} = \frac{T_s}{\mu} \times \frac{\left.\frac{\partial D}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial D}{\partial x}\right|_{x}}{\Delta x} \tag{16.27}$$

Recall, from calculus, that the derivative of the function $f(x)$ is defined as

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \left[\frac{f(x + \Delta x) - f(x)}{\Delta x} \right]$$

This is exactly what we have on the right side of Equation 16.27 if we let the width of our little piece of string approach zero: $\Delta x \to 0$. The function for which we're evaluating the difference between $x + \Delta x$ and x is the partial derivative $\partial D / \partial x$, and the derivative of a derivative is a second derivative.

Thus in the limit $\Delta x \to 0$, Equation 16.27 becomes

$$\frac{\partial^2 D}{\partial t^2} = \frac{T_s}{\mu} \frac{\partial^2 D}{\partial x^2} \quad \text{(wave equation for a string)} \tag{16.28}$$

Equation 16.28 is the *wave equation for a string*. It's really Newton's second law in disguise, but written for a continuous object where the displacement is a function of both position and time. Just like Newton's second law for a particle, it governs the dynamics of motion on a string.

Traveling Wave Solutions

The equation of motion for a simple harmonic oscillator turned out to be a second-order differential equation. Although there are systematic ways to solve differential equations, we noted that—because solutions are unique—we can sometimes use what we know about a situation to *guess* the solution. The same is true for Equation 16.28, which is a *partial differential equation*. We have reason to think that sinusoidal waves can travel on stretched strings, so let's guess that a solution to Equation 16.28 is

$$D(x, t) = A\sin(kx - \omega t + \phi_0) \tag{16.29}$$

where the minus sign gives a wave traveling in the $+x$-direction and—from Equation 16.13—the wave speed is $v = \omega/k$.

To evaluate this possible solution we need its second partial derivatives. With respect to position we have

$$\begin{aligned} \frac{\partial D}{\partial x} &= kA\cos(kx - \omega t + \phi_0) \\ \frac{\partial^2 D}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial D}{\partial x}\right) = -k^2 A\sin(kx - \omega t + \phi_0) \end{aligned} \tag{16.30}$$

and with respect to time we have

$$\begin{aligned} \frac{\partial D}{\partial t} &= -\omega A\cos(kx - \omega t + \phi_0) \\ \frac{\partial^2 D}{\partial t^2} &= \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right) = -\omega^2 A\sin(kx - \omega t + \phi_0) \end{aligned} \tag{16.31}$$

Substituting the second partial derivatives into the wave equation, Equation 16.28, gives

$$-\omega^2 A\sin(kx - \omega t + \phi_0) = \frac{T_s}{\mu}\left(-k^2 A\sin(kx - \omega t + \phi_0)\right) \tag{16.32}$$

This will be true only if

$$\omega^2 = \frac{T_s}{\mu}k^2 \tag{16.33}$$

But ω/k is the wave speed v, so what we've found is that the sinusoidal wave of Equation 16.1 *is* a solution to the wave equation, but only if the wave travels with speed

$$v = \frac{\omega}{k} = \sqrt{\frac{T_s}{\mu}} \tag{16.34}$$

You should be able to convince yourself that we would have arrived at the same result if we had started with $D(x, t) = A\sin(kx + \omega t + \phi_0)$ for a wave traveling in the $-x$-direction.

Let's summarize. We used Newton's second law for a small piece of the string to come up with an equation for the dynamics of motion on a string. We then showed that a solution to this equation is a sinusoidal traveling wave, and we made a specific prediction for the wave speed in terms of two properties or characteristics of the string: its tension and its mass density. Thus the answer to the question with which we started this section—*why* do waves propagate along a string—is that wave motion is simply a consequence of Newton's second law, the relationship between force and acceleration, when applied to a continuous object.

With Equation 16.9 in hand, we can write Equation 16.28 as

$$\frac{\partial^2 D}{\partial t^2} = v^2\frac{\partial^2 D}{\partial x^2} \quad \text{(the general wave equation)} \tag{16.35}$$

We derived Equation 16.28 specifically for a string, but *any* physical system that obeys Equation 16.35 for some type of displacement D will have sinusoidal waves traveling with speed v. Equation 16.35 is called the **wave equation,** and it occurs over and over in science and engineering. We will see it again in this chapter in our analysis of sound waves. And much later, in Chapter 31, we'll discover that electromagnetic fields also obey this equation. Thus electromagnetic waves exist, and we'll be able to predict that all electromagnetic waves, regardless of wavelength, travel through vacuum with the same speed—the speed of light.

EXAMPLE 16.4 Generating a sinusoidal wave

A very long string with $\mu = 2.0$ g/m is stretched along the x-axis with a tension of 5.0 N. At $x = 0$ m it is tied to a 100 Hz simple harmonic oscillator that vibrates perpendicular to the string with an amplitude of 2.0 mm. The oscillator is at its maximum positive displacement at $t = 0$ s.

a. Write the displacement equation for the traveling wave on the string.

b. At $t = 5.0$ ms, what is the string's displacement at a point 2.7 m from the oscillator?

MODEL The oscillator generates a sinusoidal traveling wave on a string. The displacement of the wave has to match the displacement of the oscillator at $x = 0$ m.

SOLVE a. The equation for the displacement is

$$D(x, t) = A\sin(kx - \omega t + \phi_0)$$

with A, k, ω, and ϕ_0 to be determined. The wave amplitude is the same as the amplitude of the oscillator that generates the wave, so $A = 2.0$ mm. The oscillator has its maximum displacement $y_{osc} = A = 2.0$ mm at $t = 0$ s, thus

$$D(0 \text{ m}, 0 \text{ s}) = A\sin(\phi_0) = A$$

This requires the phase constant to be $\phi_0 = \pi/2$ rad. The wave's frequency is $f = 100$ Hz, the frequency of the source; therefore the angular frequency is $\omega = 2\pi f = 200\pi$ rad/s. We still need $k = 2\pi/\lambda$, but we do not know the wavelength. However, we have enough information to determine the wave speed, and we can then use either $\lambda = v/f$ or $k = \omega/v$. The speed is

$$v = \sqrt{\frac{T_s}{\mu}} = \sqrt{\frac{5.0 \text{ N}}{0.0020 \text{ kg/m}}} = 50 \text{ m/s}$$

Using v, we find $\lambda = 0.50$ m and $k = 2\pi/\lambda = 4\pi$ rad/m. Thus the wave's displacement equation is

$$D(x, t) = (2.0 \text{ mm}) \times \sin\left[2\pi((2.0 \text{ m}^{-1})x - (100 \text{ s}^{-1})t) + \pi/2 \text{ rad}\right]$$

Notice that we have separated out the 2π. This step is not essential, but for some problems it makes subsequent steps easier.

b. The wave's displacement at $t = 5.0$ ms $= 0.0050$ s is

$$\begin{aligned} D(x, t = 5.0 \text{ ms}) &= (2.0 \text{ mm})\sin(4\pi x - \pi \text{ rad} + \pi/2 \text{ rad}) \\ &= (2.0 \text{ mm})\sin(4\pi x - \pi/2 \text{ rad}) \end{aligned}$$

At $x = 2.7$ m (calculator set to radians!), the displacement is

$$D(2.7 \text{ m}, 5.0 \text{ ms}) = 1.6 \text{ mm}$$

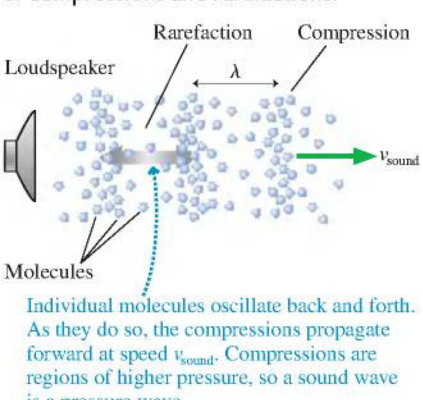
16.5 Sound and Light

Although there are many kinds of waves in nature, two are especially significant for us as humans. These are sound waves and light waves, the basis of hearing and seeing.

Sound Waves

We usually think of sound waves traveling in air, but sound can travel through any gas, through liquids, and even through solids. FIGURE 16.19 shows a loudspeaker cone vibrating back and forth in a fluid such as air or water. Each time the cone moves forward, it collides with the molecules and pushes them closer together. A half cycle later, as the cone moves backward, the fluid has room to expand and the density decreases a little. These regions of higher and lower density (and thus higher and lower pressure) are called **compressions** and **rarefactions.**

FIGURE 16.19 A sound wave is a sequence of compressions and rarefactions.



This periodic sequence of compressions and rarefactions travels outward from the loudspeaker as a longitudinal sound wave. When the wave reaches your ear, the oscillating pressure causes your eardrum to vibrate. These vibrations are transferred into your inner ear and perceived as sound.

The speed of sound waves depends on the compressibility of the medium. As TABLE 16.1 shows, the speed is faster in liquids and solids (relatively incompressible) than in gases (highly compressible). For sound waves in air, the speed at temperature T (in °C) is

$$v_{\text{sound in air}} = 331 \text{ m/s} \times \sqrt{\frac{T(°\text{C}) + 273}{273}} \tag{16.36}$$

We'll derive this result in Section 16.6, but recall from chemistry that adding 273 to a Celsius temperature converts it to an absolute temperature in kelvins. The speed of sound increases with increasing temperature but, interestingly, does *not* depend on the air pressure. For air at room temperature (20°C),

$$v_{\text{sound in air}} = 343 \text{ m/s} \qquad \text{(sound speed in air at 20°C)}$$

TABLE 16.1 The speed of sound

Medium	Speed (m/s)
Air (0°C)	331
Air (20°C)	343
Helium (0°C)	970
Ethyl alcohol	1170
Water (20°C)	1480
Granite	6000
Aluminum	6420

This is the value you should use when solving problems unless you're given temperature information.

A speed of 343 m/s is high, but not extraordinarily so. A distance as small as 100 m is enough to notice a slight delay between when you see something, such as a person hammering a nail, and when you hear it. The time required for sound to travel 1 km is $t = (1000\text{ m})/(343\text{ m/s}) \approx 3\text{ s}$. You may have learned to estimate the distance to a bolt of lightning by timing the number of seconds between when you see the flash and when you hear the thunder. Because sound takes 3 s to travel 1 km, the time divided by 3 gives the distance in kilometers. Or, in English units, the time divided by 5 gives the distance in miles.

Your ears are able to detect sound waves with frequencies between roughly 20 Hz and 20,000 Hz, or 20 kHz. You can use the fundamental relationship $v_{\text{sound}} = \lambda f$ to calculate that a 20 Hz sound wave has a 17-m-long wavelength, while the wavelength of a 20 kHz note is a mere 17 mm. Low frequencies are perceived as "low-pitch" bass notes, while high frequencies are heard as "high-pitch" treble notes. Your high-frequency range of hearing can deteriorate with age (10 kHz is the average upper limit at age 65) or as a result of exposure to very loud sounds.

Sound waves exist at frequencies well above 20 kHz, even though humans can't hear them. These are called *ultrasonic* frequencies. Oscillators vibrating at frequencies of many MHz generate the ultrasonic waves used in ultrasound medical imaging. A 3 MHz wave traveling through water (which is basically what your body is) at a sound speed of 1480 m/s has a wavelength of about 0.5 mm. It is this very small wavelength that allows ultrasound to image very small objects. We'll see why when we study *diffraction* in Chapter 33.

This ultrasound image is an example of using high-frequency sound waves to "see" within the human body.

Electromagnetic Waves

A light wave is an *electromagnetic wave,* a self-sustaining oscillation of the electromagnetic field. Other electromagnetic waves, such as radio waves, microwaves, and ultraviolet light, have the same physical characteristics as light waves even though we cannot sense them with our eyes. It is easy to demonstrate that light will pass unaffected through a container from which all the air has been removed, and light reaches us from distant stars through the vacuum of interstellar space. Such observations raise interesting but difficult questions. If light can travel through a region in which there is no matter, then what is the *medium* of a light wave? What is it that is waving?

It took scientists over 50 years, most of the 19th century, to answer this question. We will examine the answers in more detail in Chapter 31 after we introduce the ideas of electric and magnetic fields. For now we can say that light waves are a "self-sustaining oscillation of the electromagnetic field." That is, the displacement D is an electric or magnetic field. Being self-sustaining means that electromagnetic waves require *no material medium* in order to travel; hence electromagnetic waves are not mechanical waves. Fortunately, we can learn about the wave properties of light without having to understand electromagnetic fields.

It was predicted theoretically in the late 19th century, and has been subsequently confirmed, that all electromagnetic waves travel through vacuum with the same speed, called the *speed of light.* The value of the speed of light is

$$v_{\text{light}} = c = 299{,}792{,}458\text{ m/s} \qquad \text{(electromagnetic wave speed in vacuum)}$$

where the special symbol c is used to designate the speed of light. (This is the c in Einstein's famous formula $E = mc^2$.) Now *this* is really moving—about one million times faster than the speed of sound in air!

NOTE $c = 3.00 \times 10^8$ m/s is the appropriate value to use in calculations.

The wavelengths of light are extremely small. You will learn in Chapter 33 how these wavelengths are determined, but for now we will note that visible light

White light passing through a prism is spread out into a band of colors called the *visible spectrum.*

is an electromagnetic wave with a wavelength (in air) in the range of roughly 400 nm (400×10^{-9} m) to 700 nm (700×10^{-9} m). Each wavelength is perceived as a different color, with the longer wavelengths seen as orange or red light and the shorter wavelengths seen as blue or violet light. A prism is able to spread the different wavelengths apart, from which we learn that "white light" is all the colors, or wavelengths, combined. The spread of colors seen with a prism, or seen in a rainbow, is called the *visible spectrum.*

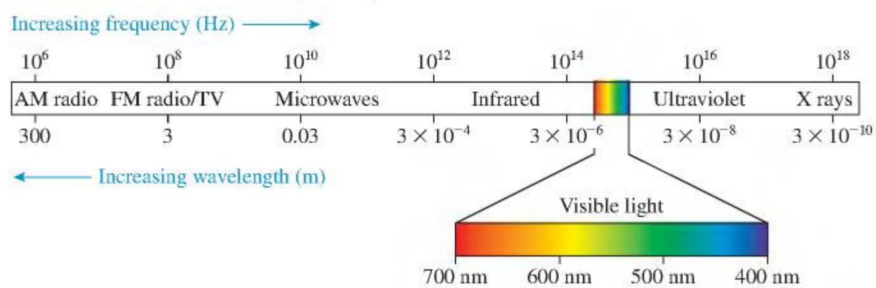
If the wavelengths of light are unbelievably small, the oscillation frequencies are unbelievably large. The frequency for a 600 nm wavelength of light (orange) is

$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{600 \times 10^{-9} \text{ m}} = 5.00 \times 10^{14} \text{ Hz}$$

The frequencies of light waves are roughly a factor of a trillion (10^{12}) higher than sound frequencies.

Electromagnetic waves exist at many frequencies other than the rather limited range that our eyes detect. One of the major technological advances of the 20th century was learning to generate and detect electromagnetic waves at many frequencies, ranging from low-frequency radio waves to the extraordinarily high frequencies of x rays. **FIGURE 16.20** shows that the visible spectrum is a small slice of the much broader **electromagnetic spectrum.**

FIGURE 16.20 The electromagnetic spectrum from 10^6 Hz to 10^{18} Hz.



EXAMPLE 16.5 Traveling at the speed of light

A satellite exploring Jupiter transmits data to the earth as a radio wave with a frequency of 200 MHz. What is the wavelength of the electromagnetic wave, and how long does it take the signal to travel 800 million kilometers from Jupiter to the earth?

SOLVE Radio waves are sinusoidal electromagnetic waves traveling with speed c. Thus

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{2.00 \times 10^8 \text{ Hz}} = 1.5 \text{ m}$$

The time needed to travel 800×10^6 km $= 8.0 \times 10^{11}$ m is

$$\Delta t = \frac{\Delta x}{c} = \frac{8.0 \times 10^{11} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 2700 \text{ s} = 45 \text{ min}$$

The Index of Refraction

Light waves travel with speed c in a vacuum, but they slow down as they pass through transparent materials such as water or glass or even, to a very slight extent, air. The slowdown is a consequence of interactions between the electromagnetic field of the wave and the electrons in the material. The speed of light in a material is characterized by the material's **index of refraction** n, defined as

$$n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in the material}} = \frac{c}{v} \qquad (16.37)$$

The index of refraction of a material is always greater than 1 because $v < c$. A vacuum has $n = 1$ exactly. **TABLE 16.2** shows the index of refraction for several materials. You can see that liquids and solids have larger indices of refraction than gases.

TABLE 16.2 Typical indices of refraction

Material	Index of refraction
Vacuum	1 exactly
Air	1.0003
Water	1.33
Glass	1.50
Diamond	2.42

NOTE An accurate value for the index of refraction of air is relevant only in very precise measurements. We will assume $n_{air} = 1.00$ in this text.

If the speed of a light wave changes as it enters into a transparent material, such as glass, what happens to the light's frequency and wavelength? Because $v = \lambda f$, either λ or f or both have to change when v changes.

As an analogy, think of a sound wave in the air as it impinges on the surface of a pool of water. As the air oscillates back and forth, it periodically pushes on the surface of the water. These pushes generate the compressions of the sound wave that continues on into the water. Because each push of the air causes one compression of the water, the frequency of the sound wave in the water must be *exactly the same* as the frequency of the sound wave in the air. In other words, **the frequency of a wave is the frequency of the source. It does not change as the wave moves from one medium to another.**

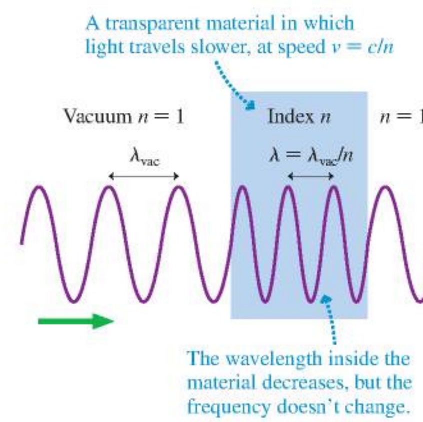
The same is true for electromagnetic waves; the frequency does not change as the wave moves from one material to another.

FIGURE 16.21 shows a light wave passing through a transparent material with index of refraction n. As the wave travels through vacuum it has wavelength λ_{vac} and frequency f_{vac} such that $\lambda_{vac} f_{vac} = c$. In the material, $\lambda_{mat} f_{mat} = v = c/n$. The frequency does not change as the wave enters ($f_{mat} = f_{vac}$), so the wavelength must. The wavelength in the material is

$$\lambda_{mat} = \frac{v}{f_{mat}} = \frac{c}{nf_{mat}} = \frac{c}{nf_{vac}} = \frac{\lambda_{vac}}{n} \qquad (16.38)$$

The wavelength in the transparent material is less than the wavelength in vacuum. This makes sense. Suppose a marching band is marching at one step per second at a speed of 1 m/s. Suddenly they slow their speed to $\frac{1}{2}$m/s but maintain their march at one step per second. The only way to go slower while marching at the same pace is to take *smaller steps.* When a light wave enters a material, the only way it can go slower while oscillating at the same frequency is to have a *smaller wavelength.*

FIGURE 16.21 Light passing through a transparent material with index of refraction n.



EXAMPLE 16.6 Light traveling through glass

Orange light with a wavelength of 600 nm is incident upon a 1.00-mm-thick glass microscope slide.

a. What is the light speed in the glass?

b. How many wavelengths of the light are inside the slide?

SOLVE a. From Table 16.2 we see that the index of refraction of glass is $n_{glass} = 1.50$. Thus the speed of light in glass is

$$v_{glass} = \frac{c}{n_{glass}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$$

b. The wavelength inside the glass is

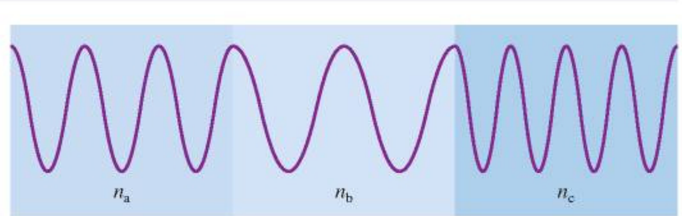
$$\lambda_{glass} = \frac{\lambda_{vac}}{n_{glass}} = \frac{600 \text{ nm}}{1.50} = 400 \text{ nm} = 4.00 \times 10^{-7} \text{ m}$$

N wavelengths span a distance $d = N\lambda$, so the number of wavelengths in $d = 1.00$ mm is

$$N = \frac{d}{\lambda} = \frac{1.00 \times 10^{-3} \text{ m}}{4.00 \times 10^{-7} \text{ m}} = 2500$$

ASSESS The fact that 2500 wavelengths fit within 1 mm shows how small the wavelengths of light are.

STOP TO THINK 16.4 A light wave travels from left to right through three transparent materials of equal thickness. Rank in order, from largest to smallest, the indices of refraction n_a, n_b, and n_c.



The Wave Model

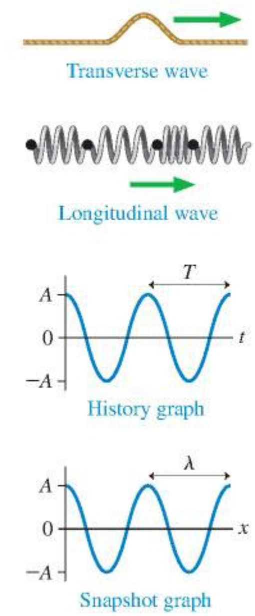
We introduced the concept of a *wave model* at the beginning of this chapter. Now we're in a position to articulate what this means.

MODEL 16.1

The wave model

A wave is an organized disturbance that travels.

- Two classes of waves:
  - **Mechanical waves** travel through a medium.
  - **Electromagnetic waves** travel through vacuum.
- Two types of waves:
  - **Transverse waves** are displaced perpendicular to the direction in which the wave travels.
  - **Longitudinal waves** are displaced parallel to the direction in which the wave travels.
- The **wave speed** is a property of the medium.
- Sinusoidal waves are periodic in both time (period) and space (wavelength).
  - The wave frequency is the oscillation frequency of the source.
  - The fundamental relationship for periodic waves is $v = \lambda f$. This says that wave moves forward one wavelength during one period.



16.6 ADVANCED TOPIC The Wave Equation in a Fluid

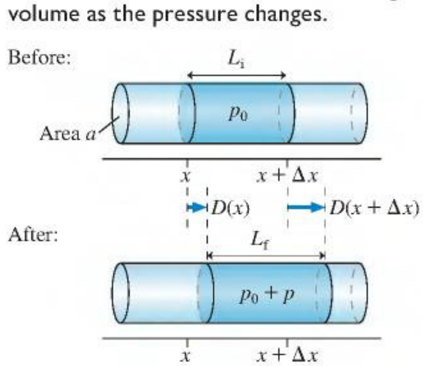
In Section 16.4 we used Newton's second law to show that traveling waves can propagate on a stretched string *and* to predict the wave speed in terms of properties of the string. Now we wish to do the same for sound waves—longitudinal waves propagating through a fluid.

A sound wave is a sequence of compressions and rarefactions in which the fluid is alternately compressed and expanded. A substance's compressibility is characterized by its *bulk modulus* B, which you met in « Section 14.6 when we looked at the elastic properties of materials. If excess pressure p is applied to an object of volume V, then the fractional change in volume—the fraction by which it's compressed—is

$$\frac{\Delta V}{V} = -\frac{p}{B} \tag{16.39}$$

The minus sign indicates that the volume *decreases* when pressure is applied. Gases are much more compressible than liquids, so gases have much smaller values of B than liquids.

FIGURE 16.22 An element of fluid changes volume as the pressure changes.



Let's apply this to a fluid—either a liquid or a gas. FIGURE 16.22 shows a small cylindrical piece of fluid with equilibrium pressure p_0 located between positions x and $x + \Delta x$. The initial length and volume of this little piece of fluid are $L_i = \Delta x$ and $V_i = aL_i = a\,\Delta x$. Notice that we use a for area in this chapter so that there is no conflict with A for amplitude. Suppose the pressure changes to $p_0 + p$. The volume of this little piece of fluid will either decrease (compression) or increase (expansion), depending on whether p is positive or negative.

The volume changes only if the ends of the cylinder undergo *different* displacements. (Equal displacements would shift the cylinder but not change its volume.) In the bottom half of Figure 16.22 we see that the left end of the cylinder has undergone displacement $D(x, t)$ while the displacement at the right end is $D(x + \Delta x, t)$. Now the cylinder has length

$$L_f = L_i + (D(x + \Delta x, t) - D(x, t)) \tag{16.40}$$

and consequently its volume has *changed* by

$$\Delta V = a(L_f - L_i) = a(D(x + \Delta x, t) - D(x, t)) \tag{16.41}$$

Substituting both the initial volume and the volume change into Equation 16.39, we have

$$\frac{\Delta V}{V} = \frac{a(D(x + \Delta x, t) - D(x, t))}{a\,\Delta x} = -\frac{p}{B} \tag{16.42}$$

After canceling the a, you can see that, just as in Section 16.4, we're left—in the limit $\Delta x \to 0$—with the definition of the derivative of D with respect to x. It is again a *partial* derivative because we're holding the variable t constant. Thus we find that the fluid pressure (or, to be exact, the pressure deviation from p_0) at position x is related to the displacement of the medium by

$$p(x, t) = -B\frac{\partial D}{\partial x} \tag{16.43}$$

The pressure depends on how rapidly the fluid's displacement changes with position.

We anticipate that we'll discover sinusoidal sound waves later in this section, so a displacement wave of amplitude A,

$$D(x, t) = A\sin(kx - \omega t + \phi_0) \tag{16.44}$$

is associated with a pressure wave

$$\begin{aligned} p(x, t) &= -B\frac{\partial D}{\partial x} = -kBA\cos(kx - \omega t + \phi_0) \\ &= -p_{max}\cos(kx - \omega t + \phi_0) \end{aligned} \tag{16.45}$$

The *pressure amplitude,* or maximum pressure, is

$$p_{max} = kBA = \frac{2\pi fBA}{v_{sound}} \tag{16.46}$$

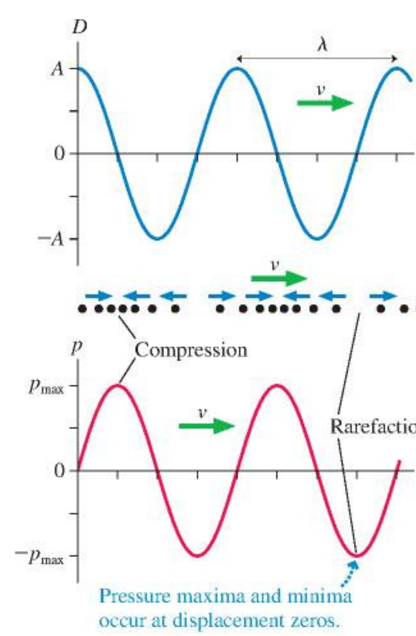
where we used $\omega = 2\pi f = vk$ (Equation 16.13) in the last step to write the result in terms of the wave's speed and frequency. In other words, a sound wave is not just a traveling wave of molecular displacement. **A sound wave is also a traveling pressure wave.**

As an example, a quite loud 100 decibel, 500 Hz sound wave in air has a pressure amplitude of 2 Pa. That is, the pressure varies around atmospheric pressure by ± 2 Pa. You can use Equation 16.46 and $B_{air} = 1.42 \times 10^5$ Pa to find that the amplitude of the oscillating air molecules is a microscopic 1.5 μm.

FIGURE 16.23 uses Equations 16.44 and 16.45 to draw snapshot graphs of displacement and pressure for a sound wave propagating to the right. Positive displacement pushes molecules to the right while negative displacement pushes them to the left, so molecules pile up (a compression) at points where the displacement is changing from positive to negative. These are the points where the displacement has the *most negative slope* and thus, from Equation 16.43, the greatest pressure.

In general, the pressure wave has a maximum or minimum at points where the displacement wave is zero, and vice versa. This observation will help us understand standing sound waves in Chapter 17.

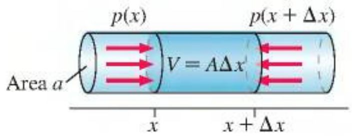
FIGURE 16.23 Snapshot graphs of the sound displacement and pressure.



Predicting the Speed of Sound

Let's return to our small, cylindrical piece of fluid and this time, in **FIGURE 16.24** on the next page, apply Newton's second law to it. The fluid pressure at position x is $p(x, t)$,

FIGURE 16.24 Fluid pressure exerts a net force on the cylinder.



and this pressure pushes the cylinder to the right with force $ap(x, t)$. At the same time, the fluid at position $x + \Delta x$ has pressure $p(x + \Delta x, t)$, and it pushes the cylinder to the left with force $ap(x + \Delta x, t)$. The net force is

$$F_{\text{net }x} = ap(x, t) - ap(x + \Delta x, t) = -a(p(x + \Delta x, t) - p(x, t)) \qquad (16.47)$$

The minus sign arises because the pressure forces push in opposite directions.

Newton's second law is $F_{\text{net }x} = ma_x$. The cylinder's mass is $m = \rho V = \rho a\,\Delta x$, where ρ is the fluid density. (Don't confuse area a with acceleration a_x!) The acceleration, just as in our analysis of a string, is the second partial derivative of displacement with respect to time. Thus the second law for our little cylinder of fluid is

$$F_{\text{net }x} = -a(p(x + \Delta x, t) - p(x, t)) = ma_x = \rho a\,\Delta x\,\frac{\partial^2 D}{\partial t^2} \qquad (16.48)$$

The area a cancels, and a slight rearrangement gives

$$\frac{\partial^2 D}{\partial t^2} = -\frac{1}{\rho}\,\frac{p(x + \Delta x, t) - p(x, t)}{\Delta x} \rightarrow -\frac{1}{\rho}\,\frac{\partial p}{\partial x} \qquad (16.49)$$

where, once again, we have a derivative in the limit $\Delta x \rightarrow 0$.

Fortunately, we already found, in Equation 16.43, that $p = -B\,\partial D/\partial x$. Substituting this for p in Equation 16.49 gives

$$\frac{\partial^2 D}{\partial t^2} = \frac{B}{\rho}\,\frac{\partial^2 D}{\partial x^2} \qquad (16.50)$$

Equation 16.50 is a wave equation! Just as in our analysis of a string, applying Newton's second law to a small piece of the medium has led to a wave equation. We've already shown that a sinusoidal traveling wave, Equation 16.44, is a solution, so we don't need to prove it again. Further, by comparing Equation 16.50 to the general wave equation, Equation 16.35, we can predict the speed of sound in a fluid:

$$v_{\text{sound}} = \sqrt{\frac{B}{\rho}} \qquad (16.51)$$

TABLE 16.3 gives values of the bulk modulus for several common fluids.

TABLE 16.3 Bulk moduli of common fluids

Medium	B (Pa)
Mercury (20°C)	2.85×10^{10}
Water (20°C)	2.18×10^{9}
Ethyl alcohol (20°C)	1.06×10^{9}
Helium (0°C, 1 atm)	1.688×10^{5}
Air (0°C, 1 atm)	1.418×10^{5}

EXAMPLE 16.7 **The speed of sound in water**

Predict the speed of sound in water at 20°C.

SOLVE From Table 16.3, the bulk modulus of water at 20°C is 2.18×10^9 Pa. The density of water is usually given as 1000 kg/m^3, but this is at 4°C. To three significant figures, the density at 20°C is 998 kg/m^3. Thus we predict

$$v_{\text{sound}} = \sqrt{\frac{2.18 \times 10^9 \text{ Pa}}{998 \text{ kg/m}^3}} = 1480 \text{ m/s}$$

This is exactly the value given earlier in Table 16.1.

For gases, both B and ρ are proportional to the pressure, so their ratio is independent of pressure. At 0°C and 1 atm, the density of air is $\rho_0 = 1.292$ kg/m^3. Thus the speed of sound in air at 0°C is

$$v_{\text{sound in air}} = \sqrt{\frac{B_0}{\rho_0}} = \sqrt{\frac{1.418 \times 10^5 \text{ Pa}}{1.292 \text{ kg/m}^3}} = 331 \text{ m/s} \qquad \text{(at 0°C)} \qquad (16.52)$$

exactly as shown in Table 16.1.

You can use the ideal-gas law to show that the density (at constant pressure) of a gas is inversely proportional to its absolute temperature T in kelvins. If the density at 0°C and 1 atm is ρ_0, then the density at temperature T is

$$\rho_T = \rho_0 \frac{273}{T(\text{K})} = \rho_0 \frac{273}{T(°\text{C}) + 273} \qquad (16.53)$$

where we used 0°C = 273 K to convert kelvin to °C. Thus a general expression for the speed of sound in air is

$$v_{\text{sound in air}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{B_0}{\rho_0}\frac{T(°C) + 273}{273}} = 331 \text{ m/s} \times \sqrt{\frac{T(°C) + 273}{273}} \tag{16.54}$$

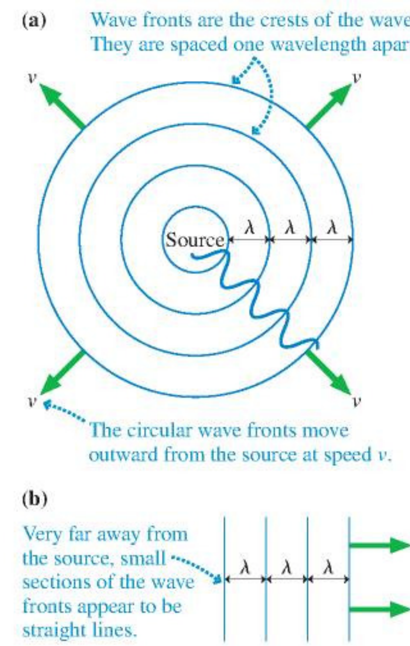
This was the expression given without proof in Section 16.5. Now we see that it comes from the wave equation for sound with a little help from the ideal-gas law.

16.7 Waves in Two and Three Dimensions

Suppose you were to take a photograph of ripples spreading on a pond. If you mark the location of the *crests* on the photo, your picture would look like FIGURE 16.25a. The lines that locate the crests are called **wave fronts,** and they are spaced precisely one wavelength apart. The diagram shows only a single instant of time, but you can imagine a movie in which you would see the wave fronts moving outward from the source at speed v. A wave like this is called a **circular wave.** It is a two-dimensional wave that spreads across a surface.

Although the wave fronts are circles, you would hardly notice the curvature if you observed a small section of the wave front very, very far away from the source. The wave fronts would appear to be parallel lines, still spaced one wavelength apart and traveling at speed v. A good example is an ocean wave reaching a beach. Ocean waves are generated by storms and wind far out at sea, hundreds or thousands of miles away. By the time they reach the beach where you are working on your tan, the crests appear to be straight lines. An aerial view of the ocean would show a wave diagram like FIGURE 16.25b.

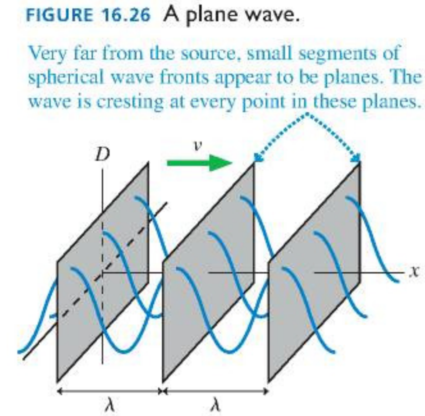
FIGURE 16.25 The wave fronts of a circular or spherical wave.



Many waves of interest, such as sound waves or light waves, move in three dimensions. For example, loudspeakers and lightbulbs emit **spherical waves.** That is, the crests of the wave form a series of concentric spherical shells separated by the wavelength λ. In essence, the waves are three-dimensional ripples. It will still be useful to draw wave-front diagrams such as Figure 16.25, but now the circles are slices through the spherical shells locating the wave crests.

If you observe a spherical wave very, very far from its source, the small piece of the wave front that you can see is a little patch on the surface of a very large sphere. If the radius of the sphere is sufficiently large, you will not notice the curvature and this little patch of the wave front appears to be a plane. FIGURE 16.26 illustrates the idea of a **plane wave.**

FIGURE 16.26 A plane wave.



To visualize a plane wave, imagine standing on the x-axis facing a sound wave as it comes toward you from a very distant loudspeaker. Sound is a longitudinal wave, so the particles of medium oscillate toward you and away from you. If you were to locate all of the particles that, at one instant of time, were at their maximum displacement toward you, they would all be located in a plane perpendicular to the travel direction. This is one of the wave fronts in Figure 16.26, and all the particles in this plane are doing exactly the same thing at that instant of time. This plane is moving toward you at speed v. There is another plane one wavelength behind it where the molecules are also at maximum displacement, yet another two wavelengths behind the first, and so on.

Because a plane wave's displacement depends on x but not on y or z, the displacement function $D(x, t)$ describes a plane wave just as readily as it does a one-dimensional wave. Once you specify a value for x, the displacement is the same at every point in the yz-plane that slices the x-axis at that value (i.e., one of the planes shown in Figure 16.26).

NOTE There are no perfect plane waves in nature, but many waves of practical interest can be modeled as plane waves.

We can describe a circular wave or a spherical wave by changing the mathematical description from $D(x, t)$ to $D(r, t)$, where r is the radial distance measured outward from the source. Then the displacement of the medium will be the same at every point on a spherical surface. In particular, a sinusoidal spherical wave with wave number k and angular frequency ω is written

$$D(r, t) = A(r)\sin(kr - \omega t + \phi_0) \tag{16.55}$$

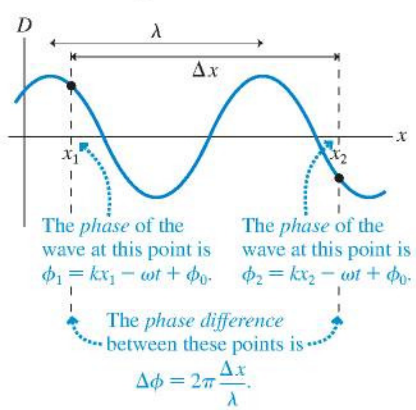
Other than the change of x to r, the only difference is that the amplitude is now a function of r. A one-dimensional wave propagates with no change in the wave amplitude. But circular and spherical waves spread out to fill larger and larger volumes of space. To conserve energy, an issue we'll look at later in the chapter, the wave's amplitude has to decrease with increasing distance r. This is why sound and light decrease in intensity as you get farther from the source. We don't need to specify exactly how the amplitude decreases with distance, but you should be aware that it does.

Phase and Phase Difference

« Section 15.2 introduced the concept of *phase* for an oscillator in simple harmonic motion. Phase is also important for waves. The **phase** of a sinusoidal wave, denoted ϕ, is the quantity $(kx - \omega t + \phi_0)$. Phase will be an important concept in Chapter 17, where we will explore the consequences of adding various waves together. For now, we can note that the wave fronts seen in Figures 16.25 and 16.26 are "surfaces of constant phase." To see this, write the displacement as simply $D(x, t) = A\sin\phi$. Because each point on a wave front has the same displacement, the phase must be the same at every point.

It will be useful to know the *phase difference* $\Delta\phi$ between two different points on a sinusoidal wave. FIGURE 16.27 shows two points on a sinusoidal wave at time t. The phase difference between these points is

FIGURE 16.27 The phase difference between two points on a wave.



$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 = (kx_2 - \omega t + \phi_0) - (kx_1 - \omega t + \phi_0)\\ &= k(x_2 - x_1) = k\,\Delta x = 2\pi\frac{\Delta x}{\lambda}\end{aligned} \tag{16.56}$$

That is, **the phase difference between two points on a wave depends on only the ratio of their separation Δx to the wavelength λ.** For example, two points on a wave separated by $\Delta x = \frac{1}{2}\lambda$ have a phase difference $\Delta\phi = \pi$ rad.

An important consequence of Equation 16.56 is that **the phase difference between two adjacent wave fronts is $\Delta\phi = 2\pi$ rad.** This follows from the fact that two adjacent wave fronts are separated by $\Delta x = \lambda$. This is an important idea. Moving from one crest of the wave to the next corresponds to changing the *distance* by λ and changing the *phase* by 2π rad.

EXAMPLE 16.8 The phase difference between two points on a sound wave

A 100 Hz sound wave travels with a wave speed of 343 m/s.

a. What is the phase difference between two points 60.0 cm apart along the direction the wave is traveling?

b. How far apart are two points whose phase differs by 90°?

MODEL Treat the wave as a plane wave traveling in the positive x-direction.

SOLVE a. The phase difference between two points is

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda}$$

In this case, $\Delta x = 60.0\text{ cm} = 0.600\text{ m}$. The wavelength is

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{100\text{ Hz}} = 3.43\text{ m}$$

and thus

$$\Delta\phi = 2\pi\frac{0.600\text{ m}}{3.43\text{ m}} = 0.350\pi\text{ rad} = 63.0°$$

b. A phase difference $\Delta\phi = 90°$ is $\pi/2$ rad. This will be the phase difference between two points when $\Delta x/\lambda = \frac{1}{4}$, or when $\Delta x = \lambda/4$. Here, with $\lambda = 3.43$ m, $\Delta x = 85.8$ cm.

ASSESS The phase difference increases as Δx increases, so we expect the answer to part b to be larger than 60 cm.

STOP TO THINK 16.5 What is the phase difference between the crest of a wave and the adjacent trough?

a. -2π rad b. 0 rad c. $\pi/4$ rad
d. $\pi/2$ rad e. π rad f. 3π rad

16.8 Power, Intensity, and Decibels

A traveling wave transfers energy from one point to another. The sound wave from a loudspeaker sets your eardrum into motion. Light waves from the sun warm the earth. The *power* of a wave is the rate, in joules per second, at which the wave transfers energy. As you learned in Chapter 9, power is measured in watts. A loudspeaker might emit 2 W of power, meaning that energy in the form of sound waves is radiated at the rate of 2 joules per second.

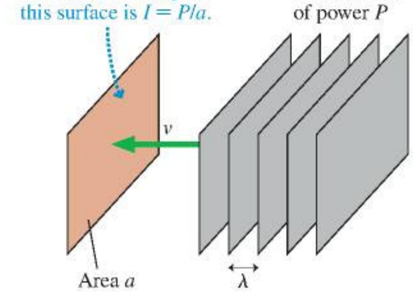
A focused light, like that of a projector, is more *intense* than the diffuse light that goes in all directions. Similarly, a loudspeaker that beams its sound forward into a small area produces a louder sound in that area than a speaker of equal power that radiates the sound in all directions. Quantities such as brightness and loudness depend not only on the rate of energy transfer, or power, but also on the *area* that receives that power.

FIGURE 16.28 shows a wave impinging on a surface of area a. The surface is perpendicular to the direction in which the wave is traveling. This might be a real, physical surface, such as your eardrum or a photovoltaic cell, but it could equally well be a mathematical surface in space that the wave passes right through. If the wave has power P, we define the **intensity** I of the wave to be

$$I = \frac{P}{a} = \text{power-to-area ratio} \tag{16.57}$$

The SI units of intensity are W/m^2. Because intensity is a power-to-area ratio, a wave focused into a small area will have a larger intensity than a wave of equal power that is spread out over a large area.

FIGURE 16.28 Plane waves of power P impinge on area a with intensity $I = P/a$.



EXAMPLE 16.9 The intensity of a laser beam

A typical red laser pointer emits 1.0 mW of light power into a 1.0-mm-diameter laser beam. What is the intensity of the laser beam?

MODEL The laser beam is a light wave.

SOLVE The light waves of the laser beam pass through a mathematical surface that is a circle of diameter 1.0 mm. The intensity of the laser beam is

$$I = \frac{P}{a} = \frac{P}{\pi r^2} = \frac{0.0010\text{ W}}{\pi(0.00050\text{ m})^2} = 1300\text{ W/m}^2$$

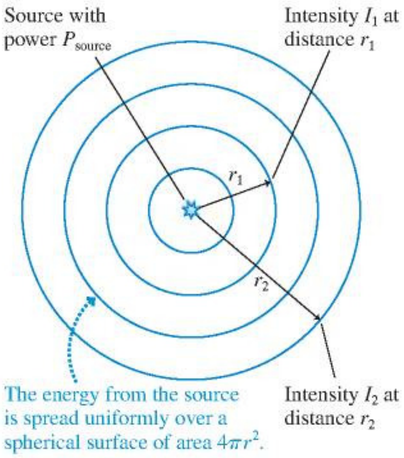
ASSESS This is roughly the intensity of sunlight at noon on a summer day. The difference between the sun and a small laser is not their intensities, which are about the same, but their powers. The laser has a small power of 1 mW. It can produce a very intense wave only because the area through which the wave passes is very small. The sun, by contrast, radiates a total power $P_{sun} \approx 4 \times 10^{26}$ W. This immense power is spread through *all* of space, producing an intensity of 1400 W/m^2 at a distance of 1.5×10^{11} m, the radius of the earth's orbit.

If a source of spherical waves radiates uniformly in all directions, then, as FIGURE 16.29 on the next page shows, the power at distance r is spread uniformly over the surface of a sphere of radius r. The surface area of a sphere is $a = 4\pi r^2$, so the intensity of a uniform spherical wave is

$$I = \frac{P_{source}}{4\pi r^2} \quad \text{(intensity of a uniform spherical wave)} \tag{16.58}$$

The inverse-square dependence of r is really just a statement of energy conservation. The source emits energy at the rate P joules per second. The energy is spread over a

FIGURE 16.29 A source emitting uniform spherical waves.



larger and larger area as the wave moves outward. Consequently, the energy *per unit area* must decrease in proportion to the surface area of a sphere.

If the intensity at distance r_1 is $I_1 = P_{source}/4\pi r_1^2$ and the intensity at r_2 is $I_2 = P_{source}/4\pi r_2^2$, then you can see that the intensity *ratio* is

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \tag{16.59}$$

You can use Equation 16.59 to compare the intensities at two distances from a source without needing to know the power of the source.

NOTE Wave intensities are strongly affected by reflections and absorption. Equations 16.58 and 16.59 apply to situations such as the light from a star or the sound from a firework exploding high in the air. Indoor sound does *not* obey a simple inverse-square law because of the many reflecting surfaces.

For a sinusoidal wave, each particle in the medium oscillates back and forth in simple harmonic motion. You learned in Chapter 15 that a particle in SHM with amplitude A has energy $E = \frac{1}{2}kA^2$, where k is the spring constant of the medium, not the wave number. It is this oscillatory energy of the medium that is transferred, particle to particle, as the wave moves through the medium.

Because a wave's intensity is proportional to the rate at which energy is transferred through the medium, and because the oscillatory energy in the medium is proportional to the *square* of the amplitude, we can infer that

$$I \propto A^2 \tag{16.60}$$

That is, **the intensity of a wave is proportional to the square of its amplitude.** If you double the amplitude of a wave, you increase its intensity by a factor of 4.

Sound Intensity Level

Human hearing spans an extremely wide range of intensities, from the *threshold of hearing* at $\approx 1 \times 10^{-12}$ W/m^2 (at midrange frequencies) to the *threshold of pain* at ≈ 10 W/m^2. If we want to make a scale of loudness, it's convenient and logical to place the zero of our scale at the threshold of hearing. To do so, we define the **sound intensity level,** expressed in **decibels** (dB), as

$$\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_0}\right) \tag{16.61}$$

where $I_0 = 1.0 \times 10^{-12}$ W/m^2. The symbol β is the Greek letter beta. Notice that β is computed as a base-10 logarithm, not a natural logarithm.

The decibel is named after Alexander Graham Bell, inventor of the telephone. Sound intensity level is actually dimensionless because it's formed from the ratio of two intensities, so decibels are just a *name* to remind us that we're dealing with an intensity *level* rather than a true intensity.

Right at the threshold of hearing, where $I = I_0$, the sound intensity level is

$$\beta = (10\text{ dB})\log_{10}\left(\frac{I_0}{I_0}\right) = (10\text{ dB})\log_{10}(1) = 0\text{ dB}$$

Note that 0 dB doesn't mean no sound; it means that, for most people, no sound is heard. Dogs have more sensitive hearing than humans, and most dogs can easily perceive a 0 dB sound. The sound intensity level at the pain threshold is

$$\beta = (10\text{ dB})\log_{10}\left(\frac{10\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right) = (10\text{ dB})\log_{10}(10^{13}) = 130\text{ dB}$$

The major point to notice is that the sound intensity level increases by 10 dB each time the actual intensity increases by a *factor* of 10. For example, the sound intensity level increases from 70 dB to 80 dB when the sound intensity increases from 10^{-5} W/m^2 to 10^{-4} W/m^2. Perception experiments find that sound is perceived as "twice as loud" when the intensity increases by a factor of 10. In terms of decibels, we can say that the perceived loudness of a sound doubles with each increase in the sound intensity level by 10 dB.

TABLE 16.4 gives the sound intensity levels for a number of sounds. Although 130 dB is the threshold of pain, quieter sounds can damage your hearing. A fairly short exposure to 120 dB can cause damage to the hair cells in the ear, but lengthy exposure to sound intensity levels of over 85 dB can produce damage as well.

TABLE 16.4 Sound intensity levels of common sounds

Sound	β (dB)
Threshold of hearing	0
Person breathing, at 3 m	10
A whisper, at 1 m	20
Quiet room	30
Outdoors, no traffic	40
Quiet restaurant	50
Normal conversation, at 1 m	60
Busy traffic	70
Vacuum cleaner, for user	80
Niagara Falls, at viewpoint	90
Snowblower, at 2 m	100
Stereo, at maximum volume	110
Rock concert	120
Threshold of pain	130
Loudest football stadium	140

EXAMPLE 16.10 Blender noise

The blender making a smoothie produces a sound intensity level of 83 dB. What is the intensity of the sound? What will the sound intensity level be if a second blender is turned on?

SOLVE We can solve Equation 16.61 for the sound intensity, finding $I = I_0 \times 10^{\beta/10\text{ dB}}$. Here we used the fact that 10 raised to a power is an "antilogarithm." In this case,

$$I = (1.0 \times 10^{-12}\text{ W/m}^2) \times 10^{8.3} = 2.0 \times 10^{-4}\text{ W/m}^2$$

A second blender doubles the sound power and thus raises the intensity to $I = 4.0 \times 10^{-4}$ W/m^2. The new sound intensity level is

$$\beta = (10\text{ dB})\log_{10}\left(\frac{4.0 \times 10^{-4}\text{ W/m}^2}{1.0 \times 10^{-12}\text{ W/m}^2}\right) = 86\text{ dB}$$

ASSESS In general, doubling the actual sound intensity increases the decibel level by 3 dB.

STOP TO THINK 16.6 Four trumpet players are playing the same note. If three of them suddenly stop, the sound intensity level decreases by

a. 40 dB b. 12 dB c. 6 dB d. 4 dB

16.9 The Doppler Effect

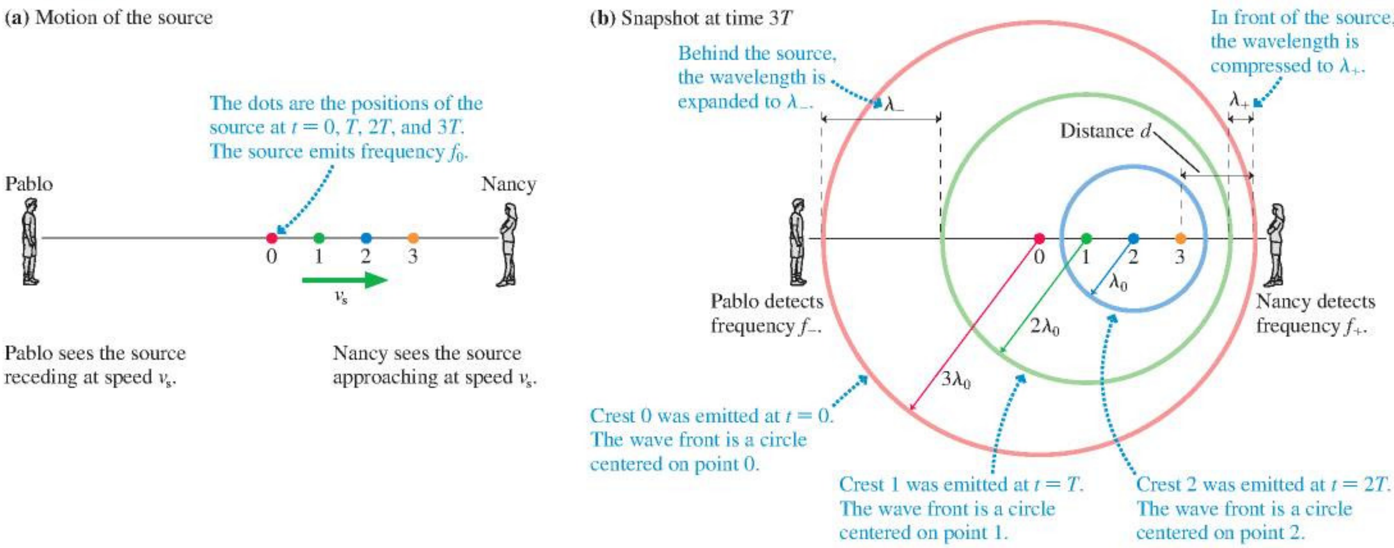
Our final topic for this chapter is an interesting effect that occurs when you are in motion relative to a wave source. It is called the *Doppler effect.* You've likely noticed that the pitch of an ambulance's siren drops as it goes past you. Why?

FIGURE 16.30a on the next page shows a source of sound waves moving away from Pablo and toward Nancy at a steady speed v_s. The subscript s indicates that this is the speed of the source, not the speed of the waves. The source is emitting sound waves of frequency f_0 as it travels. The figure is a motion diagram showing the position of the source at times $t = 0$, T, $2T$, and $3T$, where $T = 1/f_0$ is the period of the waves.

Nancy measures the frequency of the wave emitted by the *approaching source* to be f_+. At the same time, Pablo measures the frequency of the wave emitted by the *receding source* to be f_-. Our task is to relate f_+ and f_- to the source frequency f_0 and speed v_s.

After a wave crest leaves the source, its motion is governed by the properties of the medium. That is, the motion of the source cannot affect a wave that has already been emitted. Thus each circular wave front in FIGURE 16.30b is centered on the point from which it was emitted. The wave crest from point 3 was emitted just as this figure was made, but it hasn't yet had time to travel any distance.

FIGURE 16.30 A motion diagram showing the wave fronts emitted by a source as it moves to the right at speed v_s.



The wave crests are bunched up in the direction the source is moving, stretched out behind it. The distance between one crest and the next is one wavelength, so the wavelength λ_+ Nancy measures is *less* than the wavelength $\lambda_0 = v/f_0$ that would be emitted if the source were at rest. Similarly, λ_- behind the source is larger than λ_0.

These crests move through the medium at the wave speed v. Consequently, the frequency $f_+ = v/\lambda_+$ detected by the observer whom the source is approaching is *higher* than the frequency f_0 emitted by the source. Similarly, $f_- = v/\lambda_-$ detected behind the source is *lower* than frequency f_0. This change of frequency when a source moves relative to an observer is called the **Doppler effect.**

The distance labeled d in Figure 16.30b is the difference between how far the wave has moved and how far the source has moved at time $t = 3T$. These distances are

$$\Delta x_{\text{wave}} = vt = 3vT$$
$$\Delta x_{\text{source}} = v_s t = 3v_s T \tag{16.62}$$

The distance d spans three wavelengths; thus the wavelength of the wave emitted by an approaching source is

$$\lambda_+ = \frac{d}{3} = \frac{\Delta x_{\text{wave}} - \Delta x_{\text{source}}}{3} = \frac{3vT - 3v_s T}{3} = (v - v_s)T \tag{16.63}$$

You can see that our arbitrary choice of three periods was not relevant because the 3 cancels. The frequency detected in Nancy's direction is

$$f_+ = \frac{v}{\lambda_+} = \frac{v}{(v - v_s)T} = \frac{v}{(v - v_s)} f_0 \tag{16.64}$$

where $f_0 = 1/T$ is the frequency of the source and is the frequency you would detect if the source were at rest. We'll find it convenient to write the detected frequency as

$$f_+ = \frac{f_0}{1 - v_s/v} \quad \text{(Doppler effect for an approaching source)}$$
$$f_- = \frac{f_0}{1 + v_s/v} \quad \text{(Doppler effect for a receding source)} \tag{16.65}$$

Proof of the second version, for the frequency f_- of a receding source, is similar. You can see that $f_+ > f_0$ in front of the source, because the denominator is less than 1, and $f_- < f_0$ behind the source.

Doppler weather radar uses the Doppler shift of reflected radar signals to measure wind speeds and thus better gauge the severity of a storm.

EXAMPLE 16.11 **How fast are the police traveling?**

A police siren has a frequency of 550 Hz as the police car approaches you, 450 Hz after it has passed you and is receding. How fast are the police traveling? The temperature is 20°C.

MODEL The siren's frequency is altered by the Doppler effect. The frequency is f_+ as the car approaches and f_- as it moves away.

SOLVE To find v_s, we rewrite Equations 16.65 as

$$f_0 = (1 + v_s/v)f_-$$
$$f_0 = (1 - v_s/v)f_+$$

We subtract the second equation from the first, giving

$$0 = f_- - f_+ + \frac{v_s}{v}(f_- + f_+)$$

This is easily solved to give

$$v_s = \frac{f_+ - f_-}{f_+ + f_-}v = \frac{100\text{ Hz}}{1000\text{ Hz}} \times 343\text{ m/s} = 34.3\text{ m/s}$$

ASSESS If you now solve for the siren frequency when at rest, you will find $f_0 = 495$ Hz. Surprisingly, the at-rest frequency is not halfway between f_- and f_+.

A Stationary Source and a Moving Observer

Suppose the police car in Example 16.11 is at rest while you drive toward it at 34.3 m/s. You might think that this is equivalent to having the police car move toward you at 34.3 m/s, but it isn't. Mechanical waves move through a medium, and the Doppler effect depends not just on how the source and the observer move with respect to each other but also on how they move with respect to the medium. We'll omit the proof, but it's not hard to show that the frequencies heard by an observer moving at speed v_o relative to a stationary source emitting frequency f_0 are

$$f_+ = (1 + v_o/v)f_0 \quad \text{(observer approaching a source)}$$
$$f_- = (1 - v_o/v)f_0 \quad \text{(observer receding from a source)} \qquad (16.66)$$

A quick calculation shows that the frequency of the police siren as you approach it at 34.3 m/s is 545 Hz, not the 550 Hz you heard as it approached you at 34.3 m/s.

The Doppler Effect for Light Waves

The Doppler effect is observed for all types of waves, not just sound waves. If a source of light waves is receding from you, the wavelength λ_- that you detect is longer than the wavelength λ_0 emitted by the source.

Although the reason for the Doppler shift for light is the same as for sound waves, there is one fundamental difference. We derived Equations 16.65 for the Doppler-shifted frequencies by measuring the wave speed v relative to the medium. For electromagnetic waves in empty space, there is no medium. Consequently, we need to turn to Einstein's theory of relativity to determine the frequency of light waves from a moving source. The result, which we state without proof, is

$$\lambda_- = \sqrt{\frac{1 + v_s/c}{1 - v_s/c}}\,\lambda_0 \quad \text{(receding source)}$$
$$\lambda_+ = \sqrt{\frac{1 - v_s/c}{1 + v_s/c}}\,\lambda_0 \quad \text{(approaching source)} \qquad (16.67)$$

Here v_s is the speed of the source *relative to* the observer.

The light waves from a receding source are shifted to longer wavelengths $(\lambda_- > \lambda_0)$. Because the longest visible wavelengths are perceived as the color red, the light from a receding source is **red shifted.** That is *not* to say that the light is red, simply that its wavelength is shifted toward the red end of the spectrum. If $\lambda_0 = 470$ nm (blue) light emitted by a rapidly receding source is detected at $\lambda_- = 520$ nm (green), we would say that the light has been red shifted. Similarly, light from an approaching source is **blue shifted,** meaning that the detected wavelengths are shorter than the emitted wavelengths $(\lambda_+ < \lambda_0)$ and thus are shifted toward the blue end of the spectrum.

EXAMPLE 16.12 | **Measuring the velocity of a galaxy**

Hydrogen atoms in the laboratory emit red light with wavelength 656 nm. In the light from a distant galaxy, this "spectral line" is observed at 691 nm. What is the speed of this galaxy relative to the earth?

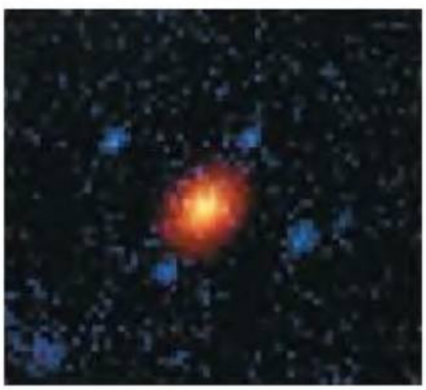
MODEL The observed wavelength is longer than the wavelength emitted by atoms at rest with respect to the observer (i.e., red shifted), so we are looking at light emitted from a galaxy that is receding from us.

SOLVE Squaring the expression for λ_- in Equations 16.67 and solving for v_s give

$$v_s = \frac{(\lambda_-/\lambda_0)^2 - 1}{(\lambda_-/\lambda_0)^2 + 1}c$$
$$= \frac{(691\text{ nm}/656\text{ nm})^2 - 1}{(691\text{ nm}/656\text{ nm})^2 + 1}c$$
$$= 0.052c = 1.56 \times 10^7\text{ m/s}$$

ASSESS The galaxy is moving away from the earth at about 5% of the speed of light!

FIGURE 16.31 A Hubble Space Telescope picture of a quasar.

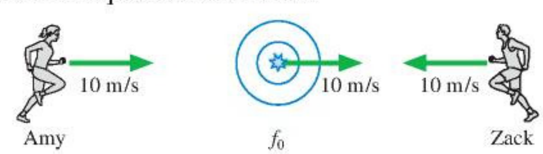


In the 1920s, an analysis of the red shifts of many galaxies led the astronomer Edwin Hubble to the conclusion that the galaxies of the universe are *all* moving apart from each other. Extrapolating backward in time must bring us to a point when all the matter of the universe—and even space itself, according to the theory of relativity—began rushing out of a primordial fireball. Many observations and measurements since have given support to the idea that the universe began in a *Big Bang* about 14 billion years ago.

As an example, FIGURE 16.31 is a Hubble Space Telescope picture of a *quasar*, short for *quasistellar object*. Quasars are extraordinarily powerful sources of light and radio waves. The light reaching us from quasars is highly red shifted, corresponding in some cases to objects that are moving away from us at greater than 90% of the speed of light. Astronomers have determined that some quasars are 10 to 12 *billion* light years away from the earth, hence the light we see was emitted when the universe was only about 25% of its present age. Today, the red shifts of distant quasars and supernovae (exploding stars) are being used to refine our understanding of the structure and evolution of the universe.

STOP TO THINK 16.7 Amy and Zack are both listening to the source of sound waves that is moving to the right. Compare the frequencies each hears.

a. $f_{Amy} > f_{Zack}$
b. $f_{Amy} = f_{Zack}$
c. $f_{Amy} < f_{Zack}$



CHALLENGE EXAMPLE 16.13 | **Decreasing the sound**

The loudspeaker on a homecoming float—mounted on a pole—is stuck playing an annoying 210 Hz tone. When the speaker is 10 m away, you measure the sound to be a loud 95 dB at 208 Hz. How long will it take for the sound intensity level to drop to a tolerable 55 dB?

MODEL The source is on a pole, so model the sound waves as uniform spherical waves. Assume a temperature of 20°C.

SOLVE The 208 Hz frequency you measure is less than the 210 Hz frequency that was emitted, so the float must be moving away from you. The Doppler effect for a receding source is

$$f_- = \frac{f_0}{1 + v_s/v}$$

We can solve this to find the speed of the float:

$$v_s = \left(\frac{f_0}{f_-} - 1\right)v = \left(\frac{210\text{ Hz}}{208\text{ Hz}} - 1\right) \times 343\text{ m/s} = 3.3\text{ m/s}$$

The sound intensity of a spherical wave decreases with the inverse square of the distance from the source. A sound intensity level β corresponds to an intensity $I = I_0 \times 10^{\beta/10\text{ dB}}$, where $I_0 = 1.0 \times 10^{-12}\text{ W/m}^2$. At the initial 95 dB, the intensity is

$$I_1 = I_0 \times 10^{9.5} = 3.2 \times 10^{-3}\text{ W/m}^2$$

At the desired 55 dB, the intensity will have dropped to

$$I_2 = I_0 \times 10^{5.5} = 3.2 \times 10^{-7}\text{ W/m}^2$$

The intensity ratio is related to the distances by

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Thus the sound will have dropped to 55 dB when the distance to the speaker is

$$r_2 = \sqrt{\frac{I_1}{I_2}}r_1 = \sqrt{10^4} \times 10\text{ m} = 1000\text{ m}$$

The float has to travel $\Delta x = 990$ m, which will take

$$\Delta t = \frac{\Delta x}{v_s} = \frac{990\text{ m}}{3.3\text{ m/s}} = 300\text{ s} = 5.0\text{ min}$$

ASSESS To drop the sound intensity level by 40 dB requires decreasing the intensity by a factor of 10^4. And with the intensity depending on the inverse square of the distance, that requires increasing the distance by a factor of 100. Floats don't move very fast—3.3 m/s is about 7 mph—so needing several minutes to travel the $\approx$ 1000 m seems reasonable.

SUMMARY

The goal of Chapter 16 has been to learn the basic properties of traveling waves.

GENERAL PRINCIPLES

The Wave Model

This model is based on the idea of a traveling wave, which is an organized disturbance traveling at a well-defined **wave speed** v.

- In transverse waves the displacement is perpendicular to the direction in which the wave travels.

- In longitudinal waves the particles of the medium are displaced parallel to the direction in which the wave travels.

A wave transfers **energy,** but no material or substance is transferred outward from the source.

Two basic classes of waves:

- Mechanical waves travel through a material medium such as water or air.
- Electromagnetic waves require no material medium and can travel through a vacuum.

For mechanical waves, such as sound waves and waves on strings, the speed of the wave is a property of the medium. Speed does not depend on the size or shape of the wave.

IMPORTANT CONCEPTS

The **displacement** D of a wave is a function of both position (where) and time (when).

- A snapshot graph shows the wave's displacement as a function of position at a single instant of time.

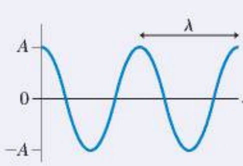
- A history graph shows the wave's displacement as a function of time at a single point in space.

For a transverse wave on a string, the snapshot graph is a picture of the wave. The displacement of a longitudinal wave is parallel to the motion; thus the snapshot graph of a longitudinal sound wave is *not* a picture of the wave.

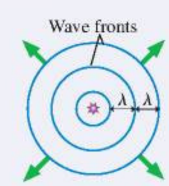
Sinusoidal waves are periodic in both time (period T) and space (wavelength λ):

$$D(x, t) = A\sin[2\pi(x/\lambda - t/T) + \phi_0]$$
$$= A\sin(kx - \omega t + \phi_0)$$

where A is the **amplitude,** $k = 2\pi/\lambda$ is the **wave number,** $\omega = 2\pi f = 2\pi/T$ is the **angular frequency,** and ϕ_0 is the **phase constant** that describes initial conditions.



One-dimensional waves



Two- and three-dimensional waves

The fundamental relationship for any sinusoidal wave is $v = \lambda f$.

APPLICATIONS

- **String** (transverse): $v = \sqrt{T_s/\mu}$
- **Sound** (longitudinal): $v = \sqrt{B/\rho} = 343$ m/s in 20°C air
- **Light** (transverse): $v = c/n$, where $c = 3.00 \times 10^8$ m/s is the speed of light in a vacuum and n is the material's **index of refraction**

The wave intensity is the power-to-area ratio: $I = P/a$

For a circular or spherical wave: $I = P_{source}/4\pi r^2$

The sound intensity level is

$$\beta = (10\text{ dB})\log_{10}(I/1.0 \times 10^{-12}\text{ W/m}^2)$$

The Doppler effect occurs when a wave source and detector are moving with respect to each other: the frequency detected differs from the frequency f_0 emitted.

Approaching source

$$f_+ = \frac{f_0}{1 - v_s/v}$$

Receding source

$$f_- = \frac{f_0}{1 + v_s/v}$$

Observer approaching a source

$$f_+ = (1 + v_o/v)f_0$$

Observer receding from a source

$$f_- = (1 - v_o/v)f_0$$

The Doppler effect for light uses a result derived from the theory of relativity.

TERMS AND NOTATION

wave model	linear density, μ	partial derivative	plane wave
traveling wave	snapshot graph	wave equation	phase, ϕ
transverse wave	history graph	compression	intensity, I
longitudinal wave	leading edge	rarefaction	sound intensity level, β
mechanical wave	trailing edge	electromagnetic spectrum	decibels
electromagnetic wave	sinusoidal wave	index of refraction, n	Doppler effect
medium	amplitude, A	wave front	red shifted
disturbance	wavelength, λ	circular wave	blue shifted
wave speed, v	wave number, k	spherical wave	

CONCEPTUAL QUESTIONS

1. The three wave pulses in FIGURE Q16.1 travel along the same stretched string. Rank in order, from largest to smallest, their wave speeds v_a, v_b, and v_c. Explain.

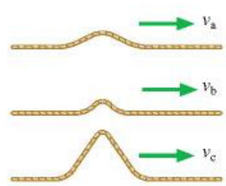


FIGURE Q16.1

2. A wave pulse travels along a stretched string at a speed of 200 cm/s. What will be the speed if:
   a. The string's tension is doubled?
   b. The string's mass is quadrupled (but its length is unchanged)?
   c. The string's length is quadrupled (but its mass is unchanged)?
   Note: Each part is independent and refers to changes made to the original string.
3. FIGURE Q16.3 is a history graph showing the displacement as a function of time at one point on a string. Did the displacement at this point reach its maximum of 2 mm *before* or *after* the interval of time when the displacement was a constant 1 mm?

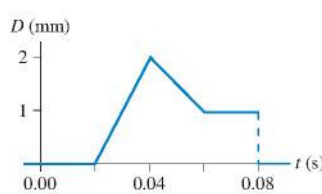


FIGURE Q16.3

4. FIGURE Q16.4 shows a snapshot graph *and* a history graph for a wave pulse on a stretched string. They describe the same wave from two perspectives.
   a. In which direction is the wave traveling? Explain.
   b. What is the speed of this wave?

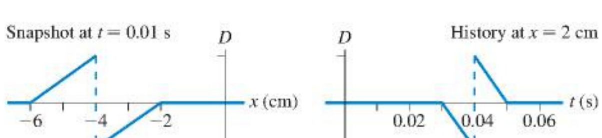


FIGURE Q16.4

5. Rank in order, from largest to smallest, the wavelengths λ_a, λ_b, and λ_c for sound waves having frequencies $f_a = 100$ Hz, $f_b = 1000$ Hz, and $f_c = 10{,}000$ Hz. Explain.
6. A sound wave with wavelength λ_0 and frequency f_0 moves into a new medium in which the speed of sound is $v_1 = 2v_0$. What are the new wavelength λ_1 and frequency f_1? Explain.
7. What are the amplitude, wavelength, frequency, and phase constant of the traveling wave in FIGURE Q16.7?

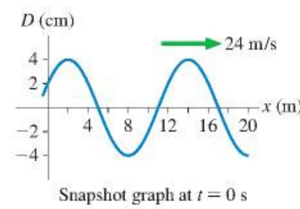


FIGURE Q16.7

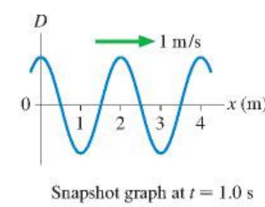


FIGURE Q16.8

8. FIGURE Q16.8 is a snapshot graph of a sinusoidal wave at $t = 1.0$ s. What is the phase constant of this wave?
9. FIGURE Q16.9 shows the wave fronts of a circular wave. What is the phase difference between (a) points A and B, (b) points C and D, and (c) points E and F?

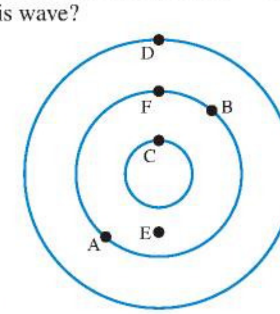


FIGURE Q16.9

10. Sound wave A delivers 2 J of energy in 2 s. Sound wave B delivers 10 J of energy in 5 s. Sound wave C delivers 2 mJ of energy in 1 ms. Rank in order, from largest to smallest, the sound powers P_A, P_B, and P_C of these three sound waves. Explain.
11. One physics professor talking produces a sound intensity level of 52 dB. It's a frightening idea, but what would be the sound intensity level of 100 physics professors talking simultaneously?
12. You are standing at $x = 0$ m, listening to a sound that is emitted at frequency f_0. The graph of FIGURE Q16.12 shows the frequency you hear during a 4-second interval. Which of the following describes the sound source? Explain your choice.
    A. It moves from left to right and passes you at $t = 2$ s.
    B. It moves from right to left and passes you at $t = 2$ s.
    C. It moves toward you but doesn't reach you. It then reverses direction at $t = 2$ s.
    D. It moves away from you until $t = 2$ s. It then reverses direction and moves toward you but doesn't reach you.

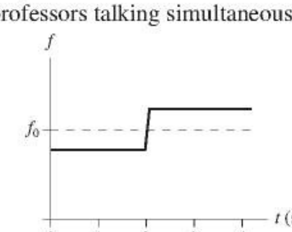


FIGURE Q16.12

EXERCISES AND PROBLEMS

Problems labeled [shaded box] integrate material from earlier chapters.

Exercises

Section 16.1 An Introduction to Waves

1. | The wave speed on a string under tension is 200 m/s. What is the speed if the tension is halved?
2. | The wave speed on a string is 150 m/s when the tension is 75 N. What tension will give a speed of 180 m/s?
3. || A 25 g string is under 20 N of tension. A pulse travels the length of the string in 50 ms. How long is the string?

Section 16.2 One-Dimensional Waves

4. || Draw the history graph $D(x = 4.0\text{ m}, t)$ at $x = 4.0$ m for the wave shown in FIGURE EX16.4.

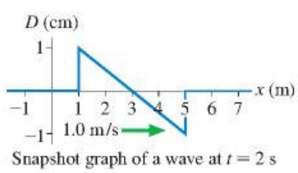


FIGURE EX16.4

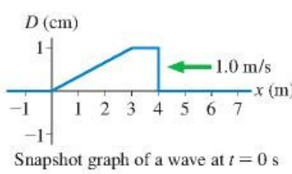


FIGURE EX16.5

5. || Draw the history graph $D(x = 0\text{ m}, t)$ at $x = 0$ m for the wave shown in FIGURE EX16.5.
6. || Draw the snapshot graph $D(x, t = 0\text{ s})$ at $t = 0$ s for the wave shown in FIGURE EX16.6.

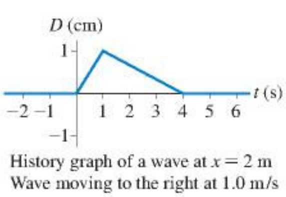


FIGURE EX16.6

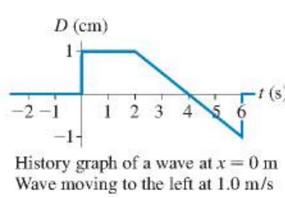


FIGURE EX16.7

7. || Draw the snapshot graph $D(x, t = 1.0\text{ s})$ at $t = 1.0$ s for the wave shown in FIGURE EX16.7.
8. || FIGURE EX16.8 is a picture at $t = 0$ s of the particles in a medium as a longitudinal wave is passing through. The equilibrium spacing between the particles is 1.0 cm. Draw the snapshot graph $D(x, t = 0\text{ s})$ of this wave at $t = 0$ s.

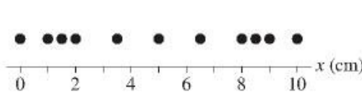


FIGURE EX16.8

9. || FIGURE EX16.9 is the snapshot graph at $t = 0$ s of a *longitudinal* wave. Draw the corresponding picture of the particle positions, as was done in Figure 16.9b. Let the equilibrium spacing between the particles be 1.0 cm.

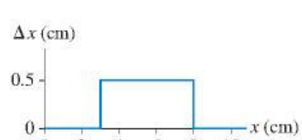


FIGURE EX16.9

Section 16.3 Sinusoidal Waves

10. | A wave has angular frequency 30 rad/s and wavelength 2.0 m. What are its (a) wave number and (b) wave speed?
11. | A wave travels with speed 200 m/s. Its wave number is 1.5 rad/m. What are its (a) wavelength and (b) frequency?
12. | The displacement of a wave traveling in the negative *y*-direction is $D(y, t) = (5.2\text{ cm})\sin(5.5y + 72t)$, where *y* is in m and *t* is in s. What are the (a) frequency, (b) wavelength, and (c) speed of this wave?
13. | The displacement of a wave traveling in the positive *x*-direction is $D(x, t) = (3.5\text{ cm})\sin(2.7x - 124t)$, where *x* is in m and *t* is in s. What are the (a) frequency, (b) wavelength, and (c) speed of this wave?
14. || What are the amplitude, frequency, and wavelength of the wave in FIGURE EX16.14?

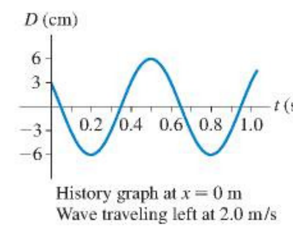


FIGURE EX16.14

Section 16.4 The Wave Equation on a String

15. || Show that the displacement $D(x, t) = cx^2 + dt^2$, where *c* and *d* are constants, is a solution to the wave equation. Then find an expression in terms of *c* and *d* for the wave speed. CALC
16. || Show that the displacement $D(x, t) = \ln(ax + bt)$, where *a* and *b* are constants, is a solution to the wave equation. Then find an expression in terms of *a* and *b* for the wave speed. CALC

Section 16.5 Sound and Light

17. || a. What is the wavelength of a 2.0 MHz ultrasound wave traveling through aluminum?
    b. What frequency of electromagnetic wave would have the same wavelength as the ultrasound wave of part a?
18. | a. What is the frequency of an electromagnetic wave with a wavelength of 20 cm?
    b. What would be the wavelength of a sound wave in water with the same frequency as the electromagnetic wave of part a?
19. | a. What is the frequency of blue light that has a wavelength of 450 nm?
    b. What is the frequency of red light that has a wavelength of 650 nm?
    c. What is the index of refraction of a material in which the red-light wavelength is 450 nm?
20. | a. An FM radio station broadcasts at a frequency of 101.3 MHz. What is the wavelength?
    b. What is the frequency of a sound source that produces the same wavelength in 20°C air?
21. | a. Telephone signals are often transmitted over long distances by microwaves. What is the frequency of microwave radiation with a wavelength of 3.0 cm?
    b. Microwave signals are beamed between two mountaintops 50 km apart. How long does it take a signal to travel from one mountaintop to the other?
22. || A hammer taps on the end of a 4.00-m-long metal bar at room temperature. A microphone at the other end of the bar picks up two pulses of sound, one that travels through the metal and one that travels through the air. The pulses are separated in time by 9.00 ms. What is the speed of sound in this metal?

23. | Cell phone conversations are transmitted by high-frequency radio waves. Suppose the signal has wavelength 35 cm while traveling through air. What are the (a) frequency and (b) wavelength as the signal travels through 3-mm-thick window glass into your room?

24. || a. How long does it take light to travel through a 3.0-mm-thick piece of window glass?
   b. Through what thickness of water could light travel in the same amount of time?

25. | A light wave has a 670 nm wavelength in air. Its wavelength in a transparent solid is 420 nm.
   a. What is the speed of light in this solid?
   b. What is the light's frequency in the solid?

26. | A 440 Hz sound wave in 20°C air propagates into the water of a swimming pool. What are the wave's (a) frequency and (b) wavelength in the water?

Section 16.6 The Wave Equation in a Fluid

27. | What is the speed of sound in air (a) on a cold winter day in Minnesota when the temperature is –25°F, and (b) on a hot summer day in Death Valley when the temperature is 125°F?

28. | The density of mercury is 13,600 kg/m^3. What is the speed of sound in mercury at 20°C?

Section 16.7 Waves in Two and Three Dimensions

29. | A circular wave travels outward from the origin. At one instant of time, the phase at $r_1 = 20$ cm is 0 rad and the phase at $r_2 = 80$ cm is 3π rad. What is the wavelength of the wave?

30. || A spherical wave with a wavelength of 2.0 m is emitted from the origin. At one instant of time, the phase at $r = 4.0$ m is π rad. At that instant, what is the phase at $r = 3.5$ m and at $r = 4.5$ m?

31. || A loudspeaker at the origin emits a 120 Hz tone on a day when the speed of sound is 340 m/s. The phase difference between two points on the x-axis is 5.5 rad. What is the distance between these two points?

32. || A sound source is located somewhere along the x-axis. Experiments show that the same wave front simultaneously reaches listeners at $x = -7.0$ m and $x = +3.0$ m.
   a. What is the x-coordinate of the source?
   b. A third listener is positioned along the positive y-axis. What is her y-coordinate if the same wave front reaches her at the same instant it does the first two listeners?

Section 16.8 Power, Intensity, and Decibels

33. || A sound wave with intensity 2.0×10^{-3} W/m^2 is perceived to
BIO be modestly loud. Your eardrum is 6.0 mm in diameter. How much energy will be transferred to your eardrum while listening to this sound for 1.0 min?

34. || The intensity of electromagnetic waves from the sun is 1.4 kW/m^2 just above the earth's atmosphere. Eighty percent of this reaches the surface at noon on a clear summer day. Suppose you think of your back as a 30 cm × 50 cm rectangle. How many joules of solar energy fall on your back as you work on your tan for 1.0 h?

35. || A concert loudspeaker suspended high above the ground emits 35 W of sound power. A small microphone with a 1.0 cm^2 area is 50 m from the speaker.
   a. What is the sound intensity at the position of the microphone?
   b. How much sound energy impinges on the microphone each second?

36. || During takeoff, the sound intensity level of a jet engine is 140 dB at a distance of 30 m. What is the sound intensity level at a distance of 1.0 km?

37. | The sun emits electromagnetic waves with a power of 4.0×10^{26} W. Determine the intensity of electromagnetic waves from the sun just outside the atmospheres of Venus, the earth, and Mars.

38. | What are the sound intensity levels for sound waves of intensity (a) 3.0×10^{-6} W/m^2 and (b) 3.0×10^{-2} W/m^2?

39. || A loudspeaker on a tall pole broadcasts sound waves equally in all directions. What is the speaker's power output if the sound intensity level is 90 dB at a distance of 20 m?

40. || The sound intensity level 5.0 m from a large power saw is 100 dB. At what distance will the sound be a more tolerable 80 dB?

Section 16.9 The Doppler Effect

41. | A friend of yours is loudly singing a single note at 400 Hz while racing toward you at 25.0 m/s on a day when the speed of sound is 340 m/s.
   a. What frequency do you hear?
   b. What frequency does your friend hear if you suddenly start singing at 400 Hz?

42. | An opera singer in a convertible sings a note at 600 Hz while cruising down the highway at 90 km/h. What is the frequency heard by
   a. A person standing beside the road in front of the car?
   b. A person on the ground behind the car?

43. || A bat locates insects by emitting ultrasonic "chirps" and then
BIO listening for echoes from the bugs. Suppose a bat chirp has a frequency of 25 kHz. How fast would the bat have to fly, and in what direction, for you to just barely be able to hear the chirp at 20 kHz?

44. || A mother hawk screeches as she dives at you. You recall from biology that female hawks screech at 800 Hz, but you hear the screech at 900 Hz. How fast is the hawk approaching?

Problems

45. || FIGURE P16.45 is a history graph at $x = 0$ m of a wave traveling in the positive x-direction at 4.0 m/s.
   a. What is the wavelength?
   b. What is the phase constant of the wave?
   c. Write the displacement equation for this wave.

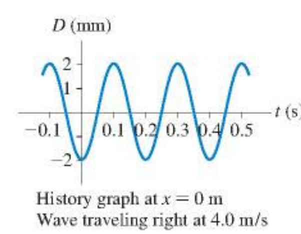


FIGURE P16.45

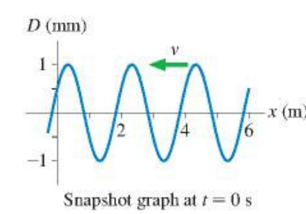


FIGURE P16.46

46. || FIGURE P16.46 is a snapshot graph at $t = 0$ s of a 5.0 Hz wave traveling to the left.
   a. What is the wave speed?
   b. What is the phase constant of the wave?
   c. Write the displacement equation for this wave.

47. | String 1 in FIGURE P16.47 has linear density 2.0 g/m and string 2 has linear density 4.0 g/m. A student sends pulses in both directions by quickly pulling up on the knot, then releasing it. What should the string lengths L_1 and L_2 be if the pulses are to reach the ends of the strings simultaneously?

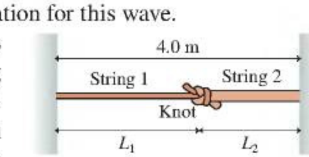


FIGURE P16.47

48. || Oil explorers set off explosives to make loud sounds, then listen for the echoes from underground oil deposits. Geologists suspect that there is oil under 500-m-deep Lake Physics. It's known that Lake Physics is carved out of a granite basin. Explorers detect a weak echo 0.94 s after exploding dynamite at the lake surface. If it's really oil, how deep will they have to drill into the granite to reach it?

49. || One cue your hearing system uses to localize a sound (i.e., to tell where a sound is coming from) is the slight difference in the arrival times of the sound at your ears. Your ears are spaced approximately 20 cm apart. Consider a sound source 5.0 m from the center of your head along a line 45° to your right. What is the difference in arrival times? Give your answer in microseconds.
BIO
Hint: You are looking for the difference between two numbers that are nearly the same. What does this near equality imply about the necessary precision during intermediate stages of the calculation?

50. || A helium-neon laser beam has a wavelength in air of 633 nm. It takes 1.38 ns for the light to travel through 30 cm of an unknown liquid. What is the wavelength of the laser beam in the liquid?

51. || Earthquakes are essentially sound waves—called seismic waves—traveling through the earth. Because the earth is solid, it can support both longitudinal and transverse seismic waves. The speed of longitudinal waves, called P waves, is 8000 m/s. Transverse waves, called S waves, travel at a slower 4500 m/s. A seismograph records the two waves from a distant earthquake. If the S wave arrives 2.0 min after the P wave, how far away was the earthquake? You can assume that the waves travel in straight lines, although actual seismic waves follow more complex routes.

52. || Helium (density 0.18 kg/m^3 at 0°C and 1 atm pressure) remains a gas until the extraordinarily low temperature of 4.2 K. What is the speed of sound in helium at 5 K?

53. || A 20.0-cm-long, 10.0-cm-diameter cylinder with a piston at one end contains 1.34 kg of an unknown liquid. Using the piston to compress the length of the liquid by 1.00 mm increases the pressure by 41.0 atm. What is the speed of sound in the liquid?

54. || A sound wave is described by $D(y, t) = (0.0200 \text{ mm}) \times \sin[(8.96 \text{ rad/m})y + (3140 \text{ rad/s})t + \pi/4 \text{ rad}]$, where y is in m and t is in s.
a. In what direction is this wave traveling?
b. Along which axis is the air oscillating?
c. What are the wavelength, the wave speed, and the period of oscillation?

55. || A wave on a string is described by $D(x, t) = (3.0 \text{ cm}) \times \sin[2\pi(x/(2.4 \text{ m}) + t/(0.20 \text{ s}) + 1)]$, where x is in m and t is in s.
a. In what direction is this wave traveling?
b. What are the wave speed, the frequency, and the wave number?
c. At $t = 0.50$ s, what is the displacement of the string at $x = 0.20$ m?

56. || A wave on a string is described by $D(x, t) = (2.00 \text{ cm}) \times \sin[(12.57 \text{ rad/m})x - (638 \text{ rad/s})t]$, where x is in m and t in s. The linear density of the string is 5.00 g/m. What are
a. The string tension?
b. The maximum displacement of a point on the string?
c. The maximum speed of a point on the string?

57. || FIGURE P16.57 shows a snapshot graph of a wave traveling to the right along a string at 45 m/s. At this instant, what is the velocity of points 1, 2, and 3 on the string?

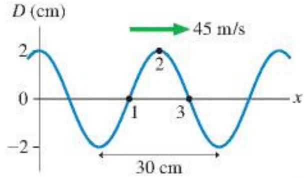


FIGURE P16.57

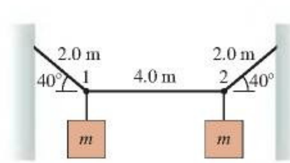


FIGURE P16.58

58. ||| FIGURE P16.58 shows two masses hanging from a steel wire. The mass of the wire is 60.0 g. A wave pulse travels along the wire from point 1 to point 2 in 24.0 ms. What is mass m?

59. || A wire is made by welding together two metals having different densities. FIGURE P16.59 shows a 2.00-m-long section of wire centered on the junction, but the wire extends much farther in both directions. The wire is placed under 2250 N tension, then a 1500 Hz wave with an amplitude of 3.00 mm is sent down the wire. How many wavelengths (complete cycles) of the wave are in this 2.00-m-long section of the wire?

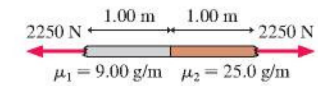


FIGURE P16.59

60. || The string in FIGURE P16.60 has linear density μ. Find an expression in terms of M, μ, and θ for the speed of waves on the string.

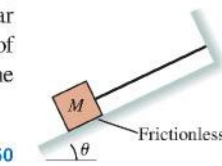


FIGURE P16.60

61. || A string that is under 50.0 N of tension has linear density 5.0 g/m. A sinusoidal wave with amplitude 3.0 cm and wavelength 2.0 m travels along the string. What is the maximum speed of a particle on the string?

62. || The G string on a guitar is a 0.46-mm-diameter steel string with a linear density of 1.3 g/m. When the string is properly tuned to 196 Hz, the wave speed on the string is 250 m/s. Tuning is done by turning the tuning screw, which slowly tightens—and stretches—the string. By how many mm does a 75-cm-long G string stretch when it's first tuned?

63. || A sinusoidal wave travels along a stretched string. A particle on the string has a maximum speed of 2.0 m/s and a maximum acceleration of 200 m/s^2. What are the frequency and amplitude of the wave?

64. ||| Is the displacement $D(x, t) = (0.10 - 0.10x^2 + xt - 2.5t^2)$ m, where x is in m and t is in s, a possible traveling wave? If so, what is the wave speed?
CALC

65. ||| Is the displacement $D(x, t) = (3.0 \text{ mm})\, e^{i(2.0x + 8.0t + 5.0)}$, where x is in m, t is in s, and $i = \sqrt{-1}$, a possible traveling wave? If so, what is the wave speed? *Complex exponentials* are often used to represent waves in more advanced treatments.
CALC

66. || An AM radio station broadcasts with a power of 25 kW at a frequency of 920 kHz. Estimate the intensity of the radio wave at a point 10 km from the broadcast antenna.

67. || LASIK eye surgery uses pulses of laser light to shave off tissue from the cornea, reshaping it. A typical LASIK laser emits a 1.0-mm-diameter laser beam with a wavelength of 193 nm. Each laser pulse lasts 15 ns and contains 1.0 mJ of light energy.
BIO
a. What is the power of one laser pulse?
b. During the very brief time of the pulse, what is the intensity of the light wave?

68. || The sound intensity 50 m from a wailing tornado siren is 0.10 W/m^2.
a. What is the intensity at 1000 m?
b. The weakest intensity likely to be heard over background noise is ≈ 1 μW/m^2. Estimate the maximum distance at which the siren can be heard.

69. || A distant star system is discovered in which a planet with twice the radius of the earth and rotating 3.0 times as fast as the earth orbits a star with a total power output of 6.8×10^{29} W.
a. If the star's radius is 6.0 times that of the sun, what is the electromagnetic wave intensity at the surface? Astronomers call this the *surface flux*. Astronomical data are provided inside the back cover of the book.
b. Every planet-day (one rotation), the planet receives 9.4×10^{22} J of energy. What is the planet's distance from its star? Give your answer in *astronomical units* (AU), where 1 AU is the distance of the earth from the sun.

70. || A compact sound source radiates 25 W of sound energy uniformly in all directions. What is the ratio of the sound intensity at a distance of 1.0 m to that at 5.0 m in (a) a two-dimensional universe, (b) our normal three-dimensional universe, and (c) a hypothetical four-dimensional universe?

71. || A loudspeaker, mounted on a tall pole, is engineered to emit 75% of its sound energy into the forward hemisphere, 25% toward the back. You measure an 85 dB sound intensity level when standing 3.5 m in front of and 2.5 m below the speaker. What is the speaker's power output?

72. || Your ears are sensitive to differences in pitch, but they are not
BIO very sensitive to differences in intensity. You are not capable of detecting a difference in sound intensity level of less than 1 dB. By what factor does the sound intensity increase if the sound intensity level increases from 60 dB to 61 dB?

73. || The intensity of a sound source is described by an inverse-square law only if the source is very small (a point source) and only if the waves can travel unimpeded in all directions. For an extended source or in a situation where obstacles absorb or reflect the waves, the intensity at distance r can often be expressed as $I = cP_{source}/r^x$, where c is a constant and the exponent x—which would be 2 for an ideal spherical wave—depends on the situation. In one such situation, you use a sound meter to measure the sound intensity level at different distances from a source, acquiring the data in the table. Use the best-fit line of an appropriate graph to determine the exponent x that characterizes this sound source.

Distance (m)	Intensity level (dB)
1	100
3	93
10	85
30	78
100	70

74. || A physics professor demonstrates the Doppler effect by tying a 600 Hz sound generator to a 1.0-m-long rope and whirling it around her head in a horizontal circle at 100 rpm. What are the highest and lowest frequencies heard by a student in the classroom?

75. || An avant-garde composer wants to use the Doppler effect in his new opera. As the soprano sings, he wants a large bat to fly toward her from the back of the stage. The bat will be outfitted with a microphone to pick up the singer's voice and a loudspeaker to rebroadcast the sound toward the audience. The composer wants the sound the audience hears from the bat to be, in musical terms, one half-step higher in frequency than the note they are hearing from the singer. Two notes a half-step apart have a frequency ratio of $2^{1/12} = 1.059$. With what speed must the bat fly toward the singer?

76. ||| A loudspeaker on a pole is radiating 100 W of sound energy in all
CALC directions. You are walking directly toward the speaker at 0.80 m/s. When you are 20 m away, what are (a) the sound intensity level and (b) the rate (dB/s) at which the sound intensity level is increasing? **Hint:** Use the chain rule and the relationship $\log_{10} x = \ln x/\ln 10$.

77. || Show that the Doppler frequency f_- of a receding source is $f_- = f_0/(1 + v_s/v)$.

78. | A starship approaches its home planet at a speed of $0.10c$. When it is 54×10^6 km away, it uses its green laser beam ($\lambda = 540$ nm) to signal its approach.
  a. How long does the signal take to travel to the home planet?
  b. At what wavelength is the signal detected on the home planet?

79. | Wavelengths of light from a distant galaxy are found to be 0.50% longer than the corresponding wavelengths measured in a terrestrial laboratory. Is the galaxy approaching or receding from the earth? At what speed?

80. | You have just been pulled over for running a red light, and the police officer has informed you that the fine will be $250. In desperation, you suddenly recall an idea that your physics professor recently discussed in class. In your calmest voice, you tell the officer that the laws of physics prevented you from knowing that the light was red. In fact, as you drove toward it, the light was Doppler shifted to where it appeared green to you. "OK," says the officer, "Then I'll ticket you for speeding. The fine is $1 for every 1 km/h over the posted speed limit of 50 km/h." How big is your fine? Use 650 nm as the wavelength of red light and 540 nm as the wavelength of green light.

Challenge Problems

81. ||| One way to monitor global warming is to measure the average temperature of the ocean. Researchers are doing this by measuring the time it takes sound pulses to travel underwater over large distances. At a depth of 1000 m, where ocean temperatures hold steady near 4°C, the average sound speed is 1480 m/s. It's known from laboratory measurements that the sound speed increases 4.0 m/s for every 1.0°C increase in temperature. In one experiment, where sounds generated near California are detected in the South Pacific, the sound waves travel 8000 km. If the smallest time change that can be reliably detected is 1.0 s, what is the smallest change in average temperature that can be measured?

82. ||| A rope of mass m and length L hangs from a ceiling.
CALC a. Show that the wave speed on the rope a distance y above the lower end is $v = \sqrt{gy}$.
  b. Show that the time for a pulse to travel the length of the string is $\Delta t = 2\sqrt{L/g}$.

83. ||| A communications truck with a 44-cm-diameter dish receiver
CALC on the roof starts out 10 km from its base station. It drives directly away from the base station at 50 km/h for 1.0 h, keeping the receiver pointed at the base station. The base station antenna broadcasts continuously with 2.5 kW of power, radiated uniformly in all directions. How much electromagnetic energy does the truck's dish receive during that 1.0 h?

84. ||| Some modern optical devic-
CALC es are made with glass whose index of refraction changes with distance from the front surface. FIGURE CP16.84 shows the index of refraction as a function of the distance into a slab of glass of thickness L. The index of refraction increases linearly from n_1 at the front surface to n_2 at the rear surface.

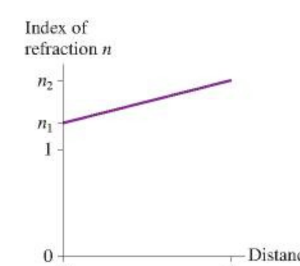


FIGURE CP16.84

  a. Find an expression for the time light takes to travel through this piece of glass.
  b. Evaluate your expression for a 1.0-cm-thick piece of glass for which $n_1 = 1.50$ and $n_2 = 1.60$.

85. ||| A water wave is a *shallow-water wave* if the water depth d is less
CALC than $\approx \lambda/10$. It is shown in hydrodynamics that the speed of a shallow-water wave is $v = \sqrt{gd}$, so waves slow down as they move into shallower water. Ocean waves, with wavelengths of typically 100 m, are shallow-water waves when the water depth is less than ≈ 10 m. Consider a beach where the depth increases linearly with distance from the shore until reaching a depth of 5.0 m at a distance of 100 m. How long does it take a wave to move the last 100 m to the shore? Assume that the waves are so small that they don't break before reaching the shore.